Web Mercator: Non-Conformal, Non-Mercator

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GIS in the Rockies
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Web Mercator Defined

- Web Mercator is the mapping of WGS84 datum (i.e. ellipsoidal) latitude / longitude into Easting / Northing using spherical Mercator equations (where R = a)
- EPSG coordinate operation method code 1024 (Popular Visualization Pseudo Mercator, PVPM)
- EPSG CRS code 3857 WGS84/PVPM (CRS code 3785 is deprecated)
- ESRI IDs 102100 (not certain about 102113)
- Safe Software (FME) 3857, 3785, 900913, LL84

John P. Snyder on Mercator

- Cylindrical
- Conformal
- Meridians are equally spaced straight lines
- Parallels are unequally spaced straight lines
- Loxodromes (rhumb lines) are straight lines
- Not perspective
- Poles are at infinity
- Used for navigation
- Presented by Mercator in 1569

Conformal Defined

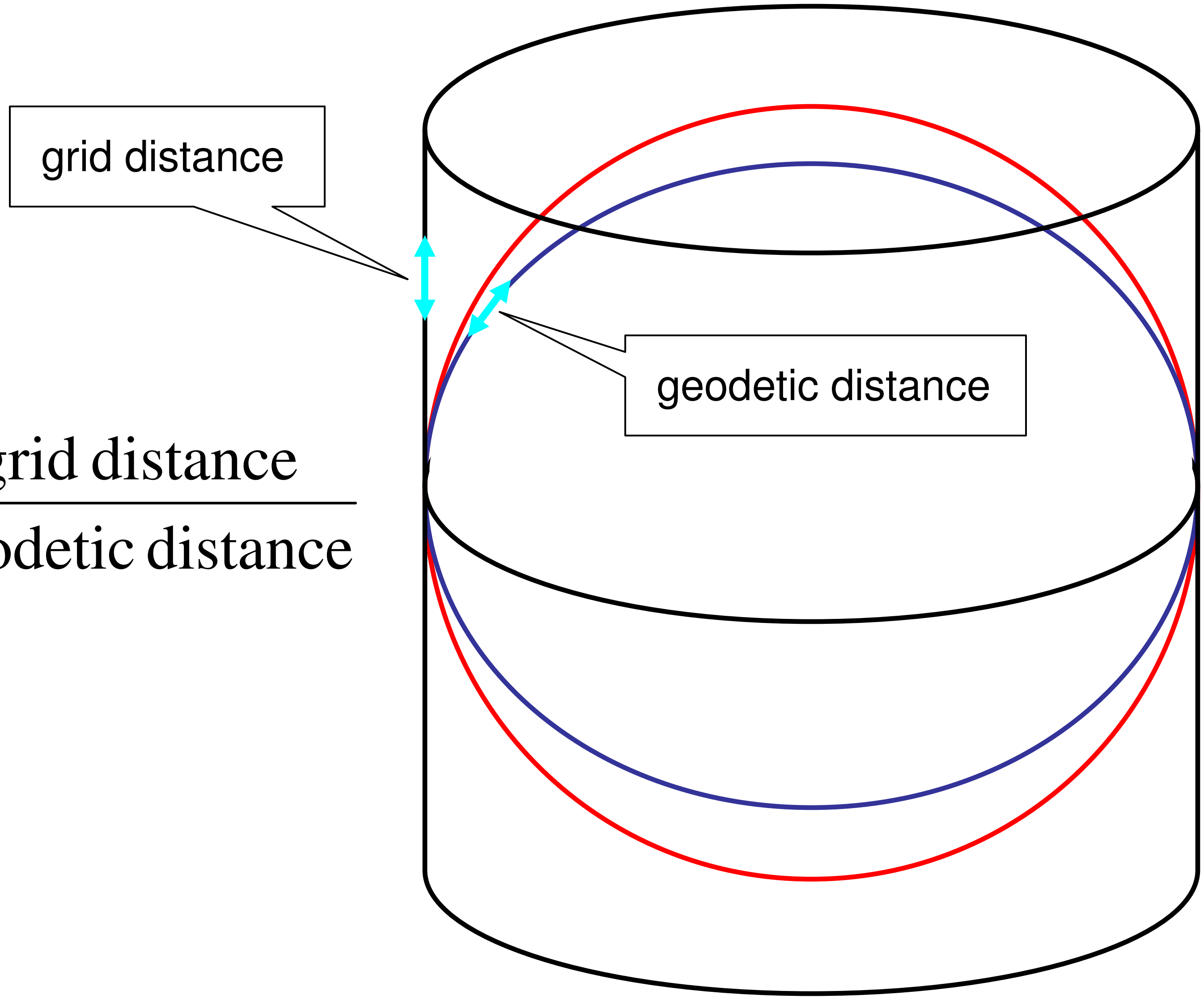
"Many of the most common and most important map projections are *conformal* or *orthomorphic* ... in that normally the shape of every *small* feature of the map is shown correctly... An important result of conformality is that relative angles at each point are correct, and the <u>local scale in every direction around any one point is constant</u>."

John P. Snyder, Map Projections Used by the U. S. Geological Survey, 1983

Point Scale Factor (k)

- On conformal maps the point scale factor (same in all directions) is the ratio of grid distance to geodetic distance
- k = grid distance / geodetic distance
- This is important for converting map distances to real-world distances quantitatively
- This applies to both the spherical and ellipsoidal Mercator
- It does not apply to the Web Mercator whose scale factor varies as a function of azimuth

Mercator Projection



$$k = \frac{\text{grid distance}}{\text{geodetic distance}}$$

Spherical Mercator Equations

$$E = R(\lambda - \lambda_0)$$ Easting

$$N = R \ln \tan(\pi / 4 + \phi / 2)$$ Northing

$$h = k = 1 / \cos \phi$$ point scale

These E/N equations and those that follow are not important. Note, however, that **h = k**, that is N/S scale factor equals E/W scale factor. This is the desired property of conformality.

Ellipsoidal Mercator Equations

$$E = a(\lambda - \lambda_0)$$ Easting

$$N = a \ln\left[\tan(\pi/4 + \phi/2)\left(\frac{1 - e\sin\phi}{1 + e\sin\phi}\right)^{e/2}\right]$$ Northing

$$h = k = (1 - e^2 \sin^2 \phi)^{1/2} / \cos\phi$$ point scale

h = k

WGS84 semi-major axis (*a*) is 6378137m

WGS84 eccentricity (*e*) is 0.081819191

Web Mercator Equations

$E = a(\lambda - \lambda_0)$ Easting

$N = a \ln \tan(\pi / 4 + \phi / 2)$ Northing

$q(\alpha) = [a / R'(\alpha)] / \cos\phi$ scale(α)

Where,

$R'(\alpha) = \rho\nu / \left(\nu \cos^2 \alpha + \rho \sin^2 \alpha\right)$ radius(α)

$\rho = a(1 - e^2) / (1 - e^2 \sin^2 \phi)^{3/2}$ radius meridian

$\nu = a / (1 - e^2 \sin^2 \phi)^{1/2}$ radius prime vertical

$h = q(0,180) = (a / \rho) / \cos\phi$ scale N/S

$k = q(90,270) = (a / \nu) / \cos\phi$ scale E/W

h <> k, therefore non-conformal

Computational Efficiency

Relative times for 1M conversions based on
spherical forward without scale (black)
spherical reverse without scale (blue)

FORWARD	
Spherical E/N only	1
Spherical E/N + scale	2.3
Ellipsoidal E/N only	4.9
Ellipsoidal E/N + scale	9
Web E/N only	1
Web E/N + scale	12.4

REVERSE		
Spherical L/L only	2.7	1
Spherical L/L + scale	4	1.5
Ellipsoidal L/L only	14.3	5.3
Ellipsoidal L/L + scale	15.8	5.9
Web L/L only	2.9	1.1
Web L/L + scale	13.9	5.2

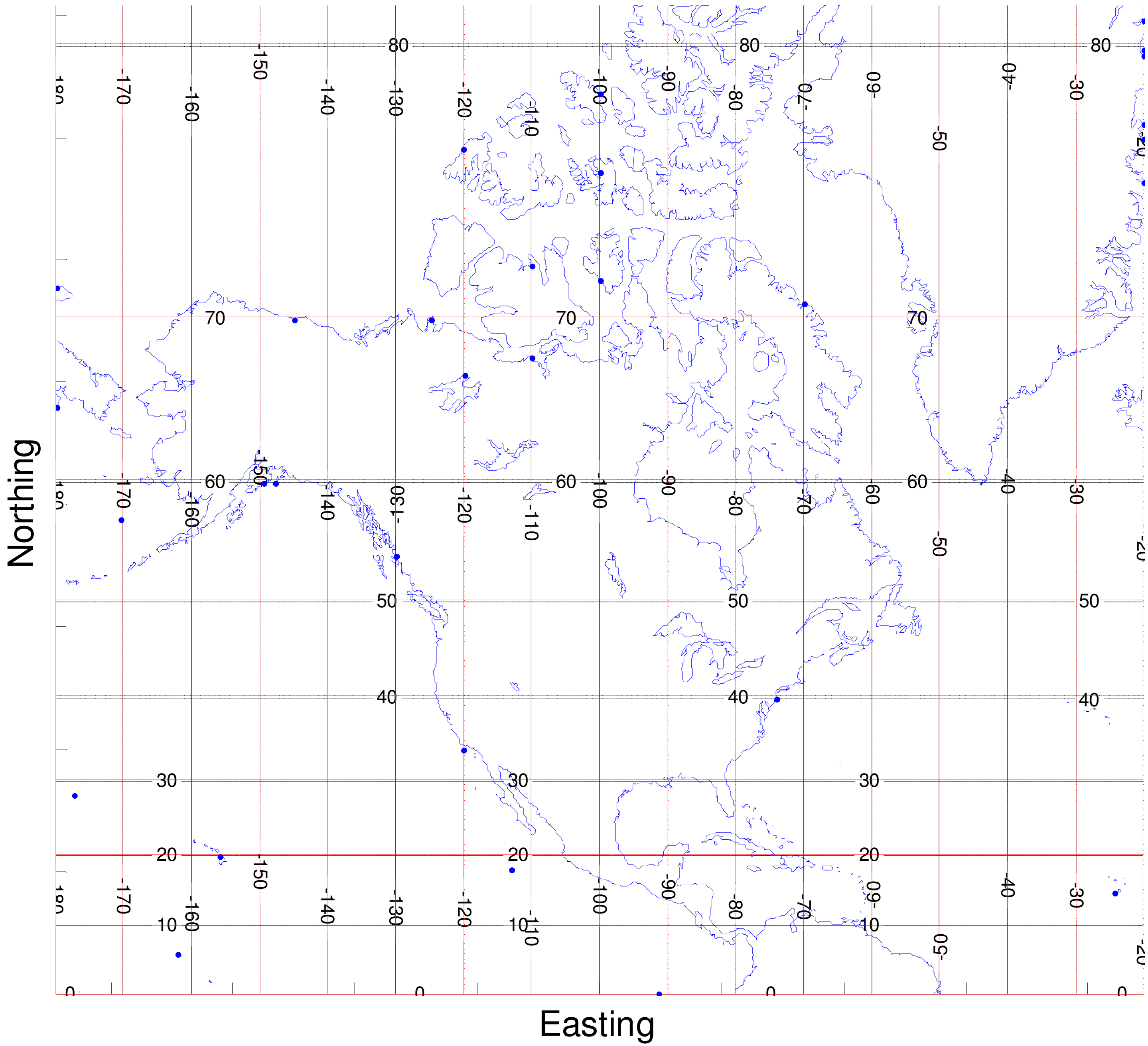
Web Mercator is not faster if scale is computed

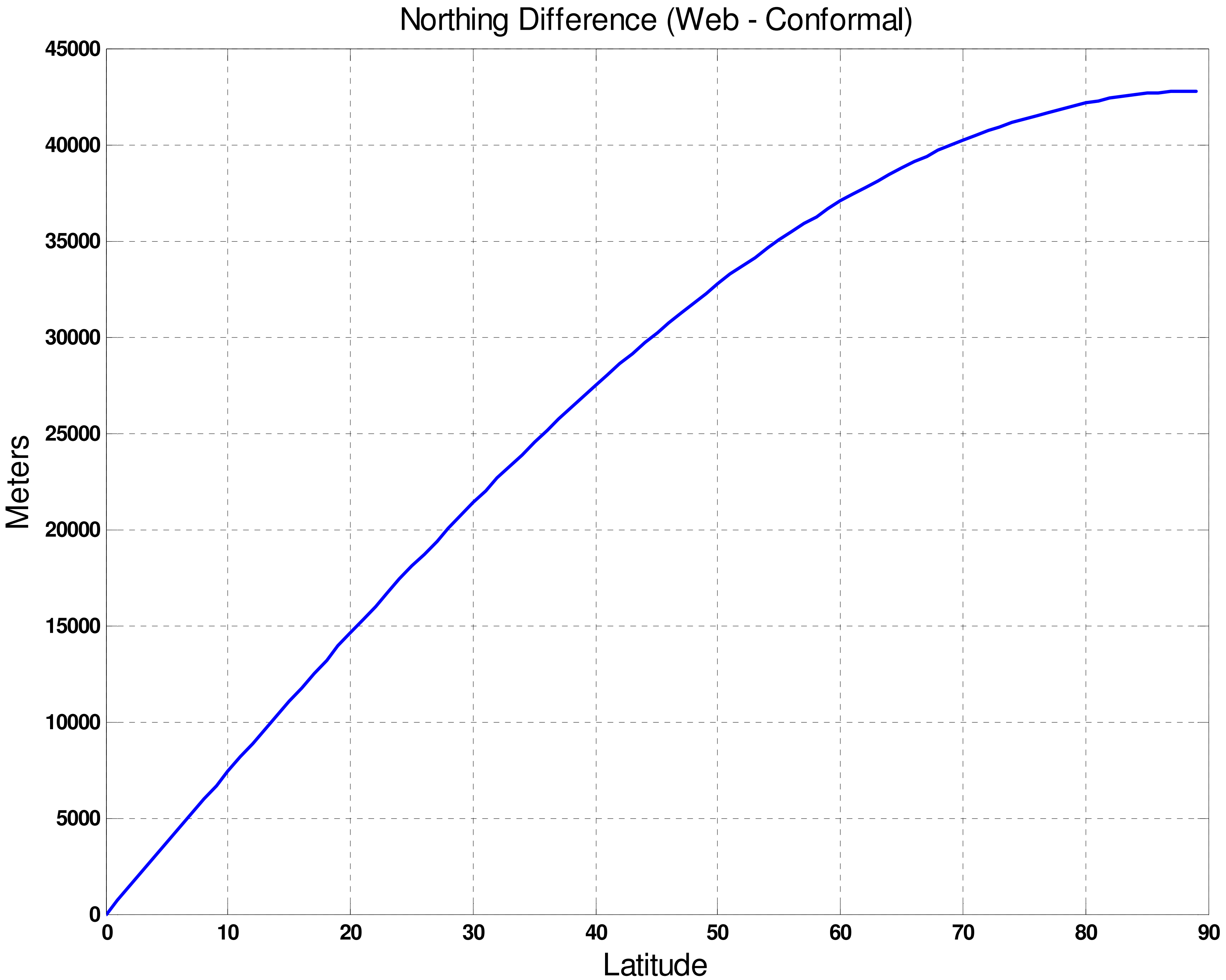
So what?!?

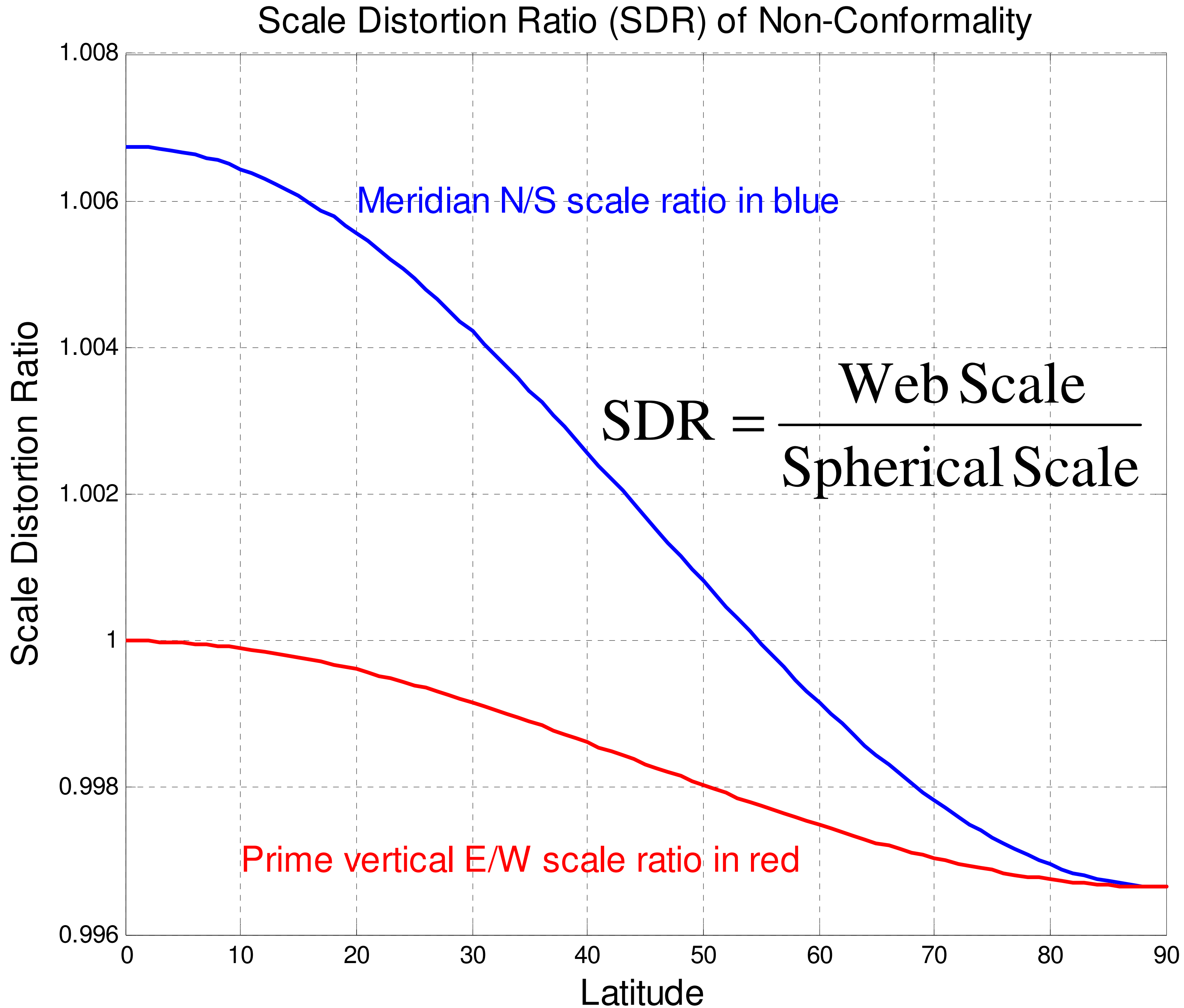
The Web Mercator is just for visualization on the web!

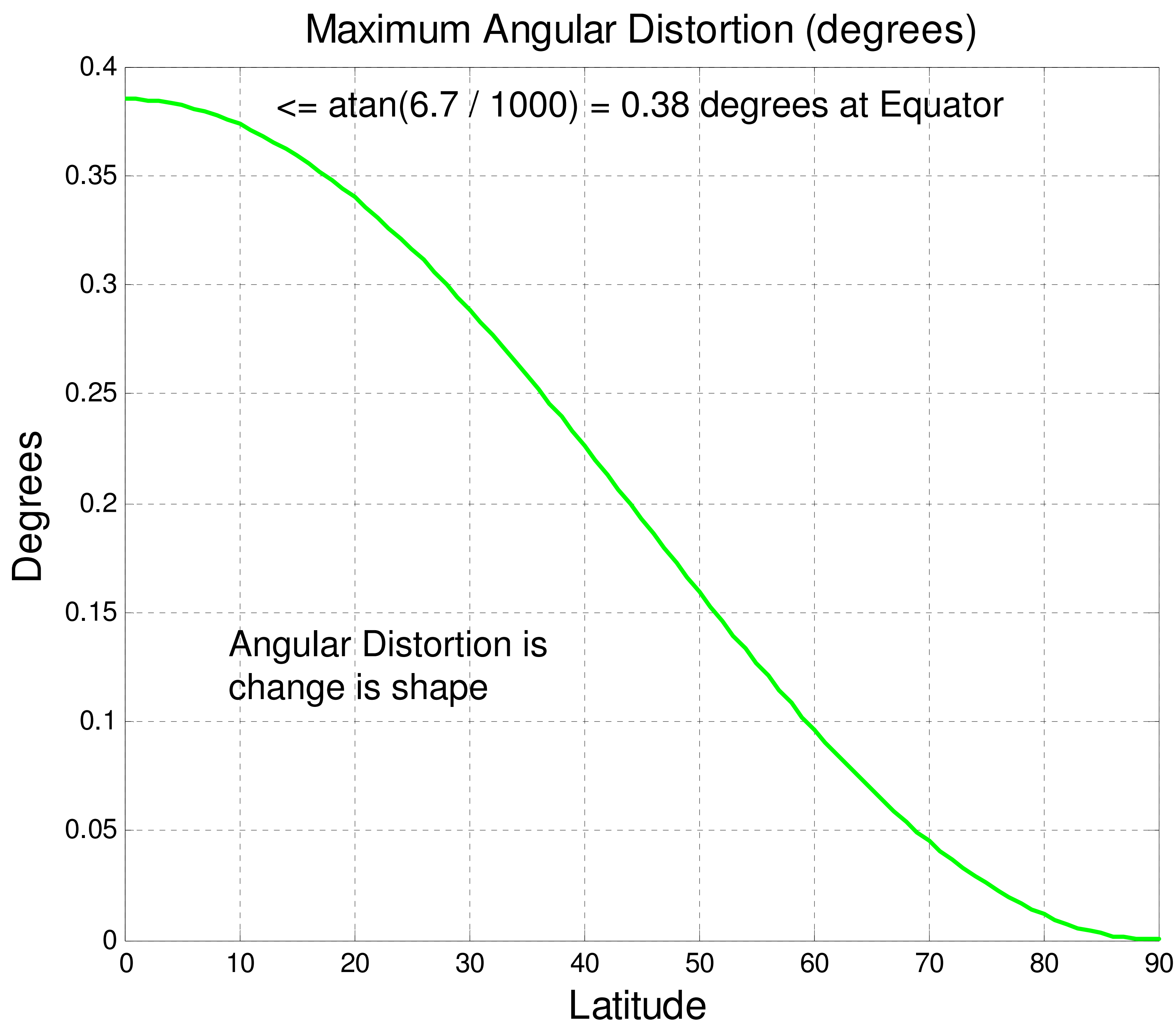
Or is it?

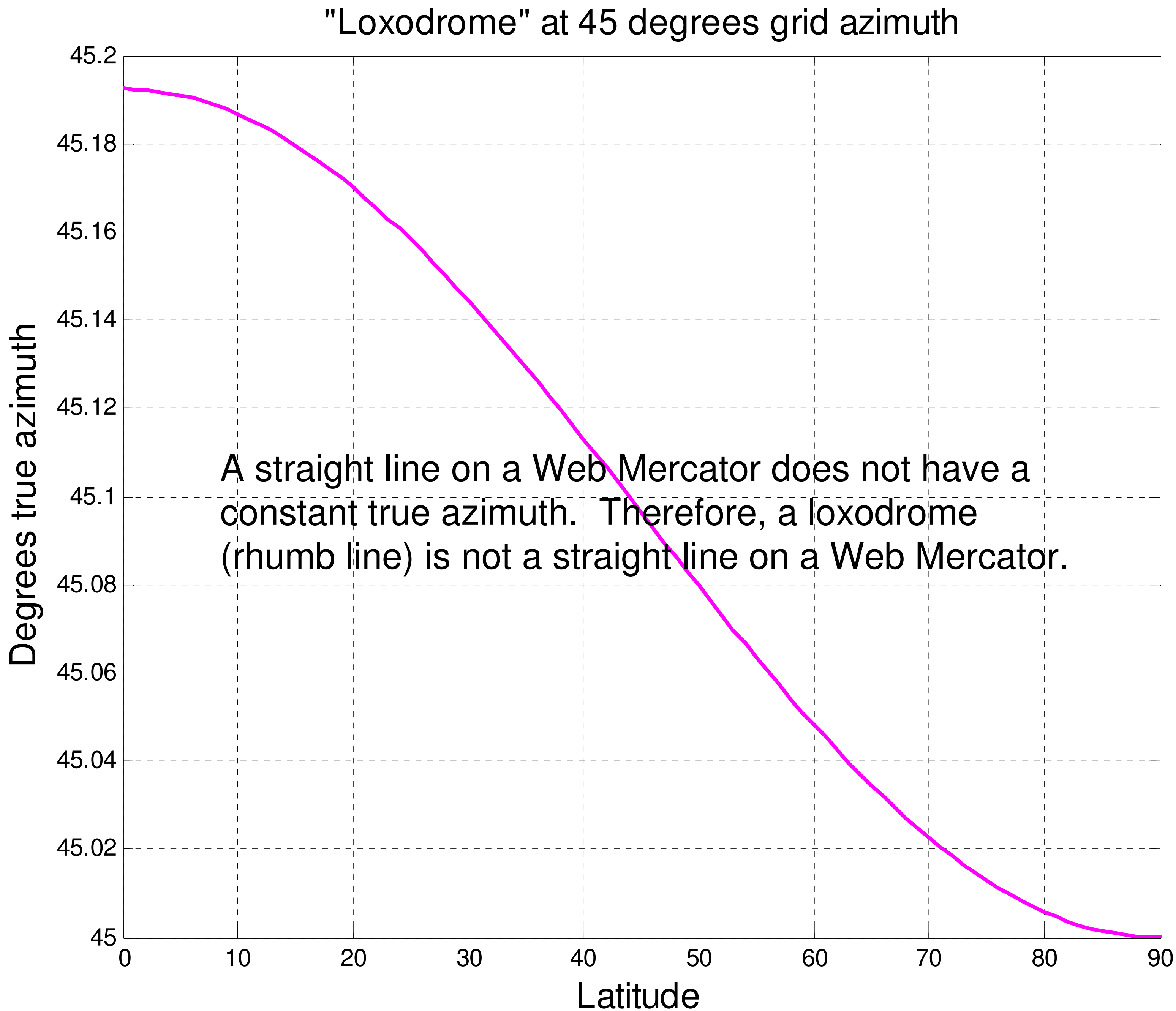
Conformal Mercator Graticule (black) / Web Mercator Graticule (red)




Northing Difference (Web - Conformal)
Meters
Latitude
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0
10
20
30
40
50
60
70
80
90


Scale Distortion Ratio (SDR) of Non-Conformality
Meridian N/S scale ratio in blue
$$SDR = \frac{\text{Web Scale}}{\text{Spherical Scale}}$$
Prime vertical E/W scale ratio in red
Scale Distortion Ratio
Latitude
1.008
1.006
1.004
1.002
1
0.998
0.996
0
10
20
30
40
50
60
70
80
90


Maximum Angular Distortion (degrees)
<= atan(6.7 / 1000) = 0.38 degrees at Equator
Angular Distortion is
change is shape
Degrees
Latitude
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60
70
80
90


"Loxodrome" at 45 degrees grid azimuth
Degrees true azimuth
45.2
45.18
45.16
45.14
45.12
45.1
45.08
45.06
45.04
45.02
45
0
10
20
30
40
50
60
70
80
90
Latitude
A straight line on a Web Mercator does not have a constant true azimuth. Therefore, a loxodrome (rhumb line) is not a straight line on a Web Mercator.

Web Mercator: True / False

- Cylindrical (yes)
- Conformal (no)
- Meridians are equally spaced straight lines (yes)
- Parallels are unequally spaced straight lines (yes, but in a different way than a conformal Mercator)
- Loxodromes (rhumb lines) are straight lines (no)
- Not perspective (yes, not perspective; mathematical)
- Poles are at infinity (yes)
- Used for navigation (doubtful)
- Presented by Mercator in 1569 (no, presented by Google in the 21st century)

Concluding Syllogism

Mercator projection is conformal.

Web Mercator is non-conformal.

Therefore, Web Mercator is not a Mercator projection.

Corollary:

If you need to use the Web Mercator quantitatively (i.e. other than for visualization), you’ll pay a price for its deceptive simplicity

References

- EPSG Guidance Note 7, Part 2, Section 1.3.3.2
  - www.epsg.org
- Discussion on the Proj4 listserver with great contributions by Mikael Rittri and daan Strebe (who have no responsibility for anything I say)
  - http://lists.maptools.org/pipermail/proj/
- This paper with speaker's note is available for download at the author's website
  - www.hydrometronics.com

Appendix

Reverse Spherical Mercator

$$\lambda = E / R + \lambda_0$$ longitude

$$\phi = \pi / 2 - 2\arctan(\exp(-N / R))$$ latitude

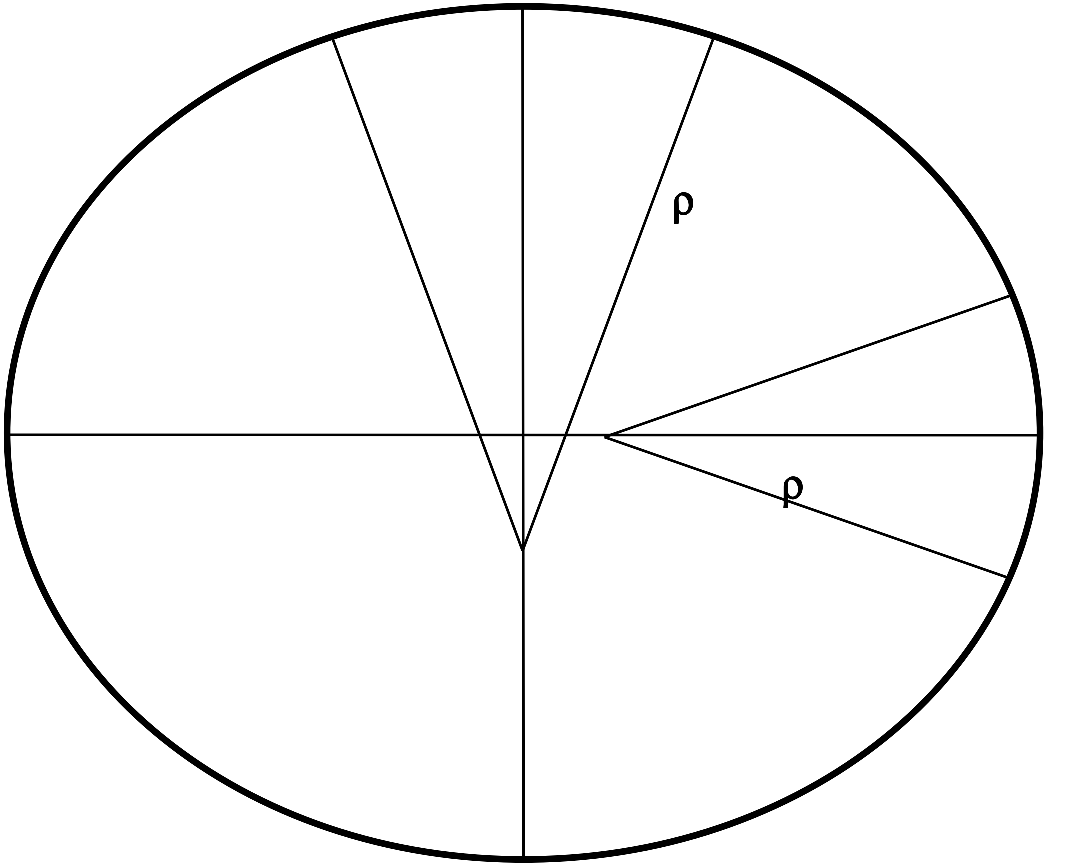
Reverse Ellipsoidal Mercator

$$\lambda = E / a + \lambda_0$$

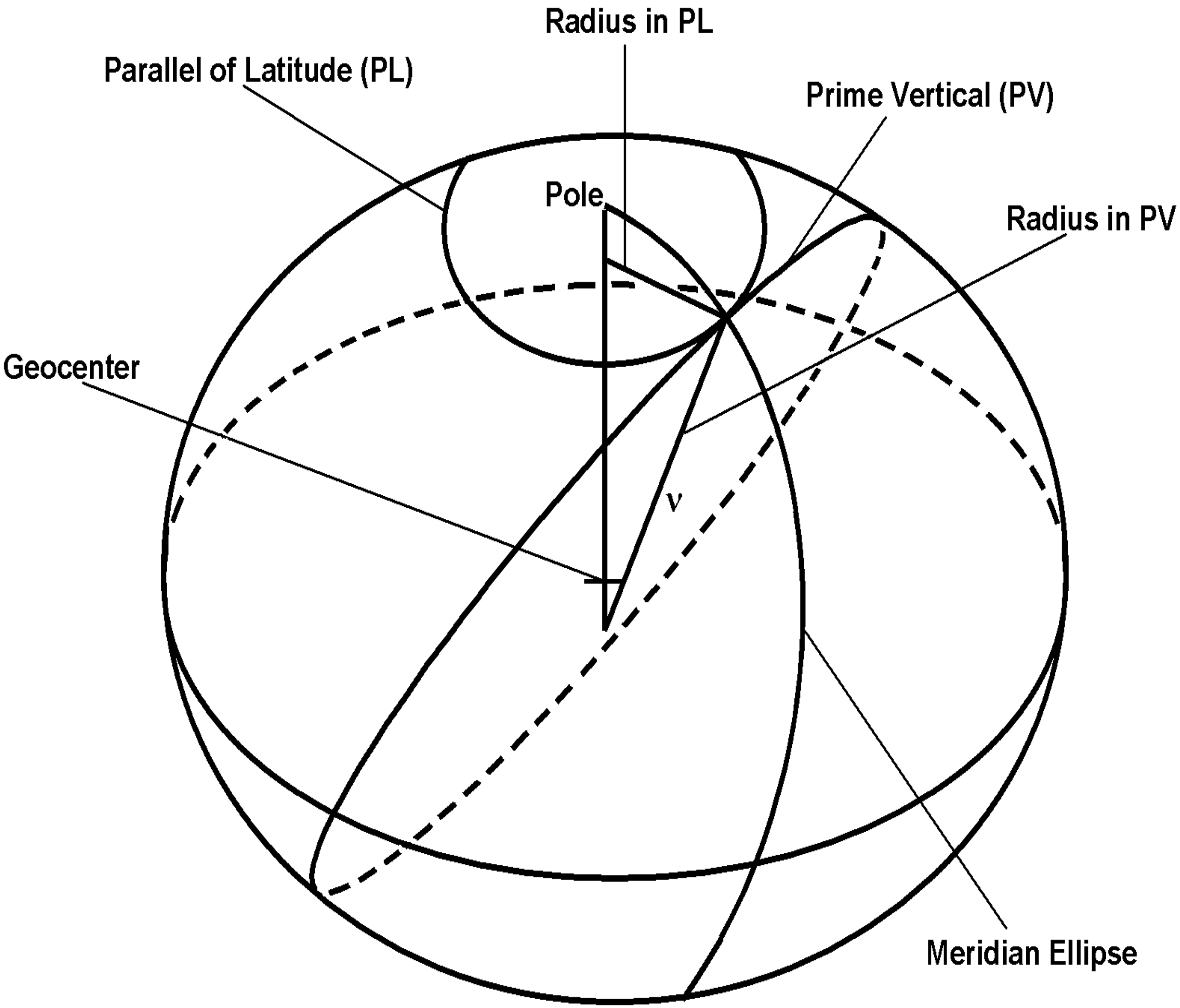
$$\phi = \pi / 2 - 2\arctan\left\{\exp(-N / a)\left[\frac{(1 - e\sin\phi)}{(1 + e\sin\phi)}\right]^{e/2}\right\}$$

Latitude is solved iteratively. Seed with spherical.

Radii in the Meridian



Radius in Prime Vertical



Mini Bio of Noel Zinn

- Noel Zinn began Hydrometronics LLC this year as a technical software consultancy
- Geodesist for ExxonMobil in the Naughts
- Navigation Scientist for Western Geophysical in the Nineties
- Surveyor for NCS International in the Eighties
- Navigator for Delta Exploration (Singapore) in the Seventies
- Peace Corps Volunteer in India in the Sixties
- Studied at the University of California (Berkeley) and the University of Houston (still there!)

Hydrometronics LLC

Hydrometronics provides consultancy and technical software development for seismic navigation, ocean-bottom positioning, subsea survey, geodesy, cartography, 3D visualization (ECEF) and wellbore-trajectory computation.

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