Horizontal Midpoint (HMP) Accuracy in Marine Seismic

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Summary

Survey pre-analysis is as useful for determining the accuracy of marine seismic assets (vessel, streamer, guns, tail buoy) as it is for fixing those in land surveys. Marine assets (targets) are connected by a variety of survey observations: gyrocompass azimuths, cable compass azimuths, acoustic ranges, laser ranges, angular measurements, streamer section lengths between sensors, and latitude/longitude devices such as GPS. These observations have reasonably well-known errors. Survey pre-analysis converts observational errors into positional errors. Survey pre-analysis lets us quantify the relative benefits of different, prospective deployments of navigation sensors.

The horizontal midpoint (HMP) is itself not a target, but is the average of the positions of a gun target and a streamer target. In this paper I apply Gauss's law of covariance propagation to elements of the pre-analysis (or actual) covariance matrix to determine HMP error expressed as variances and a covariance. Additionally, I demonstrate that the HMP inline sigma (the square root of the inline variance) is at most the average of the inline sigmas of the respective gun and source targets and can be as little as zero. The same can be demonstrated for the crossline sigma. Actual marine seismic configurations are pre-analyzed and target and HMP accuracies are given.

Blunders, Biases and Random Errors

A navigational blunder is a discrete, "far out" error such as a data "spike" due to a signal bounce or a dropped bit. A bias is a smaller, systematic error such as might be introduced by a poorly calibrated ranging device. The analysis in this report assumes the absence of blunder and bias in all navigation observations. It is the responsibility of one's navigation experts to identify, quantify and eliminate blunder and bias from the navigation solution. Random errors can be identified, can be decreased as technology improves and their effects minimized by good geometry; but random errors cannot be eliminated. In this report I analyze the impact of random error on target accuracy. Careless navigation will always produce worse results than those modeled herein.

Statistical accuracy and normally-distributed random error are such related concepts that the terms are often interchanged. That a blunder-free, unbiased acoustic range is accurate to ±1 meter at 95% confidence, for example, is implied by the knowledge that its random error or sigma (the square root of its variance) is 0.5 meters.

Survey Pre-analysis

The least-squares technique of variation of coordinates is a best, linear, unbiased estimator (BLUE) of station coordinates. Nowadays least squares is always weighted; that is, observations are given weights based on their quality. Good measurements contribute more to a solution than do poor measurements. An observation's weight is the inverse of its random variance, known either ***a priori*** (e.g., manufacturer's specification) or ***a posteriori*** (by monitoring recent trends in the data). The variation of coordinates technique propagates observation errors into station position errors. The benefit is that we can know the accuracy of our station coordinates if we know the accuracy of our observations.

Least squares is intended to adjust real observations among a network of real stations. We can, however, short-circuit this real data requirement by adjusting artificial observations which exactly fit our predetermined station coordinates. Still, the variation of coordinates algorithm will propagate typical observation errors into typical station errors. This short-circuiting technique is called pre-analysis and is the approach taken in this paper to determine the accuracies of marine seismic targets. Pre-analysis of different marine configurations is easily performed with the appropriate software.

Dynamic navigation differs from static surveying in that stations are moving targets, variation of coordinates is often replaced by a Kalman filter, and observations occur at irregular intervals. It is important to recognize that the Kalman filter can actually be derived from a least squares approach (see Krakiwsky, 1975). Although a Kalman filter will contain transition equations and observations between states not accommodated by the pre-analysis of this paper, most observations will be treated the same, even using the same observation equations. Survey pre-analysis is as close as we can come to a snap-shot of achievable navigation accuracies without throwing our hands up in despair over an infinitude of possibilities.

Gauss's Law of Covariance Propagation

Consider the matrix equation

$$p = A \cdot q, \tag{1}$$

where p and q are stochastic vectors and A is their deterministic relationship. If C_q is the covariance matrix of q, then the covariance matrix of p (C_p) can be determined by the relationship

$$C_p = A \cdot C_q \cdot A^T, \tag{2}$$

This relationship is succinctly proven by Cross (1983).

HMP Accuracy

The HMP is the mean (X_m, Y_m) of two target positions: a source (X_s, Y_s) and a receiver (X_r, Y_r) where X is taken to be the inline coordinate and Y the crossline. HMP is represented by the matrix equation (1) where

$$p = \begin{bmatrix} X_m \\ Y_m \end{bmatrix}, \tag{3}$$

$$A = \begin{bmatrix} .5 & 0 & .5 & 0 \\ 0 & .5 & 0 & .5 \end{bmatrix}, \text{ and} \tag{4}$$

$$q = \begin{bmatrix} X_s \\ Y_s \\ X_r \\ Y_r \end{bmatrix}. \tag{5}$$

The covariance matrix of q is

$$C_q = \begin{bmatrix} \sigma^2_{X_s} & \sigma_{X_s Y_s} & \sigma_{X_s X_r} & \sigma_{X_s Y_r} \\ \sigma_{X_s Y_s} & \sigma^2_{Y_s} & \sigma_{X_r Y_s} & \sigma_{Y_s Y_r} \\ \sigma_{X_s X_r} & \sigma_{X_r Y_s} & \sigma^2_{X_r} & \sigma_{X_r Y_r} \\ \sigma_{X_s Y_r} & \sigma_{Y_s Y_r} & \sigma_{X_r Y_r} & \sigma^2_{Y_r} \end{bmatrix}, \tag{6}$$

where the elements of C_q are elements of the larger covariance matrix of the network adjustment rearranged compactly. These elements may be determined in pre-analysis or in the adjustment of actual data by least squares or by a Kalman filter.

Now, by applying Gauss's law of covariance propagation we find that the covariance matrix of the mean position (C_p) is given by equation (2). The form of this matrix is

$$C_p = \begin{bmatrix} \sigma^2_{X_m} & \sigma_{X_m Y_m} \\ \sigma_{X_m Y_m} & \sigma^2_{Y_m} \end{bmatrix}. \tag{7}$$

where

$$\sigma^2_{X_m} = \left(\sigma^2_{X_s} + 2 \cdot \sigma_{X_s X_r} + \sigma^2_{X_r}\right) / 4 , \tag{8}$$

$$\sigma^2_{Y_m} = \left(\sigma^2_{Y_s} + 2 \cdot \sigma_{Y_s Y_r} + \sigma^2_{Y_r}\right) / 4 , \tag{9}$$

$$\sigma_{X_m Y_m} = \left(\sigma_{X_s Y_s} + \sigma_{X_s Y_r} + \sigma_{X_r Y_r} + \sigma_{X_r Y_s}\right) / 4. \tag{10}$$

The terms of equations (8), (9) and (10) define the accuracy (or random error) of the HMP between the source and receiver targets.

Upper and Lower Bounds of HMP Inline and Crossline Error

Correlation is a measure of the statistical dependence between two stochastic variables and varies between 1 and -1. If X and Y are stochastic variables, their correlation (ρ) is defined by the relationship

$$\rho = \sigma_{XY} / \left(\sigma_X \cdot \sigma_Y\right). \tag{11}$$

The covariance term (σ_{XY}) will take on the values of $(\sigma_X \cdot \sigma_Y)$, 0 and $-(\sigma_X \cdot \sigma_Y)$ as ρ varies from 1 to 0 to -1. With this knowledge and with equations (8) and (9) above, we can compute the upper and lower bounds of σ_{X_m} and σ_{Y_m} in terms of σ_{X_s}, σ_{X_r}, σ_{Y_s} and σ_{Y_r}. We will do this for σ_{X_m} only, since σ_{Y_m} follows similarly.

Repeating equation (8), we have

$$\sigma^2_{X_m} = \left(\sigma^2_{X_s} + 2 \cdot \sigma_{X_s X_r} + \sigma^2_{X_r}\right) / 4 . \tag{8}$$

When $\rho = 1$, equation (8) becomes

$$\begin{aligned} \sigma^2_{X_m} &= \left(\sigma^2_{X_s} + 2 \cdot \sigma_{X_s} \cdot \sigma_{X_r} + \sigma^2_{X_r}\right) / 4 \\ &= \left(\sigma_{X_s} + \sigma_{X_r}\right)^2 / 4 \end{aligned} \tag{12}$$

or

$$\sigma_{X_m} = \left(\sigma_{X_s} + \sigma_{X_r}\right) / 2. \tag{13}$$

In this case, σ_{X_m} is the mean of σ_{X_s} and σ_{X_r}.

When $\rho = -1$, equation (8) gives

$$\begin{aligned} \sigma^2_{X_m} &= \left(\sigma^2_{X_s} - 2 \cdot \sigma_{X_s} \cdot \sigma_{X_r} + \sigma^2_{X_r}\right) / 4 \\ &= \left(\sigma_{X_s} - \sigma_{X_r}\right)^2 / 4, \end{aligned} \tag{14}$$

or

$$\sigma_{X_m} = \left|\sigma_{X_s} - \sigma_{X_r}\right| / 2, \tag{15}$$

which can be 0 if $\sigma_{X_s} = \sigma_{X_r}$; σ_{X_m} will always be less than the mean of σ_{X_s} and σ_{X_r}.

When $\rho = 0$, perhaps a more typical case,

$$\sigma^2_{X_m} = \left(\sigma^2_{X_s} + \sigma^2_{X_r}\right) / 4, \tag{16}$$

or

$$\sigma_{X_m} = \sqrt{\left(\sigma^2_{X_s} + \sigma^2_{X_r}\right)} / 2. \tag{17}$$

σ_{X_m} is also always less than the mean value, since if we increase the $\sigma^2_{X_m}$ of equation (16) by $(\sigma_{X_s} \cdot \sigma_{X_r} / 2)$, we get equation (12), which is the mean.

Modeled Configurations

In the dual-cable, dual-source configurations pre-analyzed in this paper, the following one sigma errors are assumed.

Vessel navigation:	2.0 meters in X/Y (uncorrelated)
Buoy navigation:	3.0 meters in X/Y (uncorrelated)
Gyro compass:	0.7 degrees
Cable compasses:	0.5 degrees
Cable section length:	0.1 meters
All acoustic ranges:	0.2 meters ± 1000 ppm, or 0.5 meters
Tail rope compass:	6.0 degrees
Tail rope length:	3.0 meters

It is important to note that the cables are 4800 meters with 12 evenly distributed compasses, the cable separation is 200 meters with a 300 meter step-back, the source separation is 100 meters with a 150 meter step-back, the tail buoy lead-in ropes are 100 meters, the front-end acoustic positioning system hull receiver baseline is 34 meters and the fore hull receiver is taken to be the navigation antenna position. These geometrical factors also affect error propagation.

The configurations consist of the following navigation options:

(1) Front-end acoustics and cable compasses,
(2) Option (1) plus tail buoys,
(3) Option (2) plus tail acoustics,
(4) Option (3) plus head acoustics,
(5) Option (4) plus sparse cable acoustics,
(6) Option (5) plus network cable acoustics,
(7) Option (3) plus head buoy,
(8) Option (7) plus sparse cable acoustics,
(9) Option (8) plus network cable acoustics.

Front-end acoustics consist of hull-mounted receivers, near-vessel pingers and pinger ranges to the guns and streamer heads. Tail acoustics are ranges in the configuration of a braced quadrilateral connecting the tail buoys to the end of the cable. Head acoustics are a braced quadrilateral of ranges between the head of the cables and the head of the stretch section including compass constraints. A head buoy is a navigation buoy, similar to a tail buoy, towed between the guns and the heads of the cables. Sparse cable acoustics are cross-cable ranges at 1200, 2400, and 3600 meters. Network cable acoustics are connected braced quadrilaterals every 400 meters from the head to the end of the cable.

Results

Of all the data that could be reported, only the poorest (that is, the least accurate) source and cable target (receiver) inline and crossline sigmas and the inline and crossline sigmas of the connecting HMP are given below. Covariance terms are not given. Numbers are in meters.

	Source		Receiver		HMP	
Option	in	cross	in	cross	in	cross
1	2.4	5.2	3.1	14.6	2.0	7.7
2	2.0	4.3	2.4	8.7	1.5	4.8
3	1.8	3.9	2.0	6.5	1.3	3.8
4	1.6	2.3	1.6	6.3	1.1	3.3
5	1.6	2.3	1.6	5.0	1.1	2.8
6	1.6	2.0	1.5	2.6	1.1	1.6
7	1.4	2.0	1.7	6.3	1.1	3.3
8	1.4	2.0	1.7	5.0	1.1	2.7
9	1.4	1.8	1.4	2.5	1.0	1.5

Conclusions

As expected, increasingly more elaborate deployments of navigational sensors do decrease target errors. It is also seen that the HMP error is always less than the worse connecting target error and, in some configurations, is less than either of the connecting target errors. This unintuitive result has been demonstrated mathematically. The HMP may be a more geophysically-meaningful index of navigational efficacy than target accuracy.

Acknowledgments

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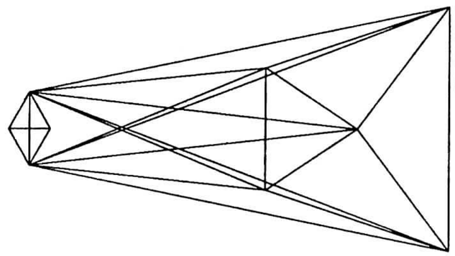
References

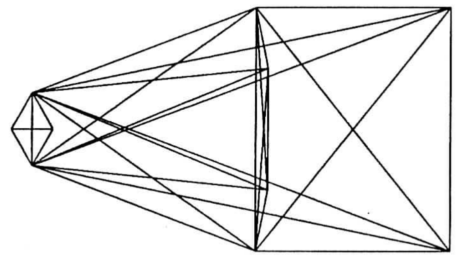
Cross, P.A., 1983, Advanced least squares applied to position-fixing: North East London Polytechnic.

Krakiwsky, E.J., 1975, A synthesis of recent advances in the method of least squares: University of New Brunswick.



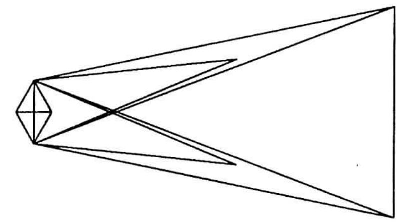
Front-end Acoustics plus

Head Buoy with Acoustics



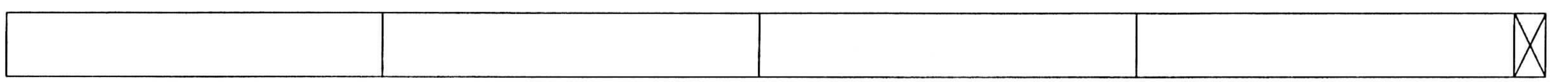
Front-end Acoustics plus

Head Acoustics



Front-end Acoustics

Sparse Cable Acoustics with Tail Acoustics



Network Cable Acoustics with Tail Acoustics