Earth Centered Earth Fixed
Powered by Blue Marble

Noel Zinn

Hydrometronics LLC

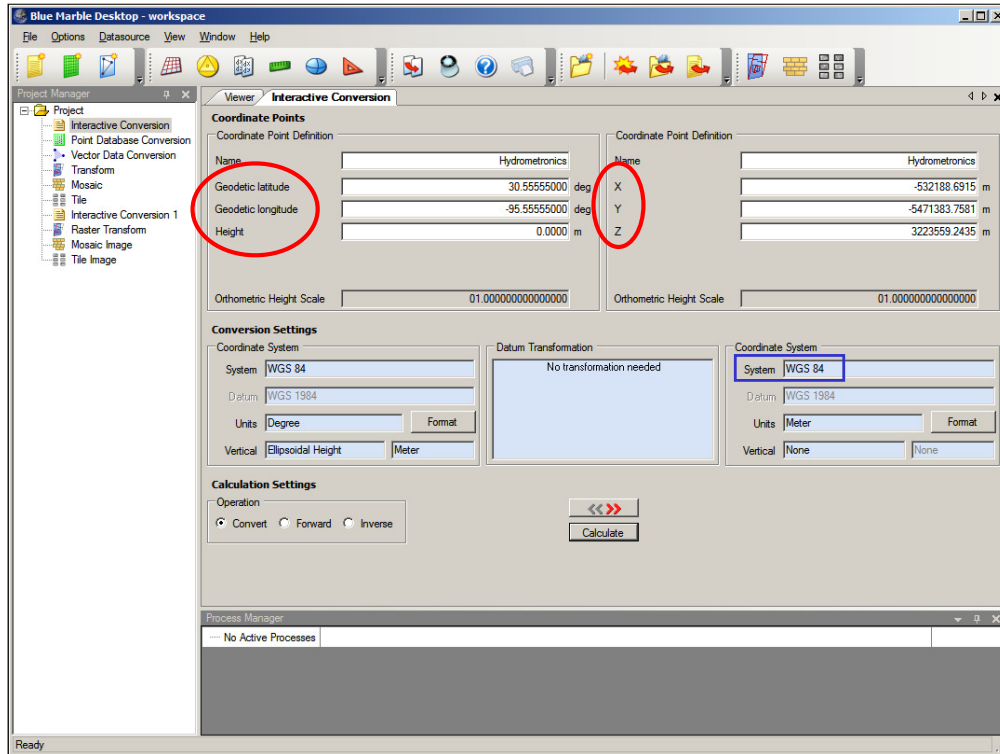Blue Marble User's Conference

October 2010

My talk this afternoon is about an Earth-Centered Earth-Fixed scheme for geodetically rigorous, 3D visualization. This scheme can be powered by Blue Marble Geographic Calculator.

Later on in the talk I'll provide a URL from which you can download this presentation.

To be clear about the crux of this scheme I present here a capture from the Blue Marble Geographic Calculator (BMGC) showing WGS84 geographical (or geodetic) coordinates on the left (latitude / longitude / height) converted to WGS84 geocentric coordinates on the right (X / Y / Z with respect to the geocenter). The location happens to be the mailbox in front of my home office.

Geocentric Cartesian coordinates and Earth-Centered Earth-Fixed are one and the same thing.

Now if I were to double click in the WGS84 Coordinate System box on the right I get the next slide.
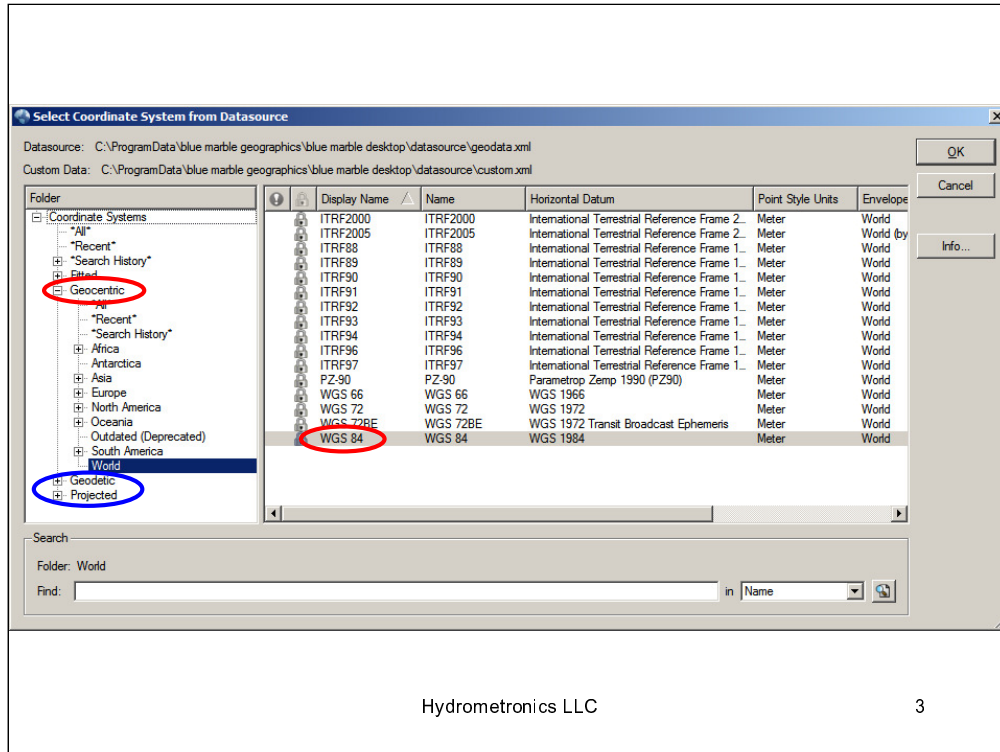
So, here's another BMGC capture that explains more closely the selection of Geocentric coordinates in WGS84.

On the left of the screen we have some other choices. They are Geodetic and Projected circled in blue. The change from one Coordinate System – or CS - representation of a point to another (among the choices of Geocentric, Geodetic or Projected) is called a conversion in that it is mathematically exact to within the numerical precision of the algorithm and the computer used.

On the other hand, the change from one datum to another (loosely called Coordinate Reference System - or CRS - in EPSG speak) is called a transformation because the transformation parameters are empirically derived and, therefore, inexact. Just approximations.

Today's talk is about conversion among coordinate systems and not transformations among coordinate reference systems.

If this vocabulary is unfamiliar to you, I hope that much of it will be explained as my talk proceeds.

Version 2.00 1993
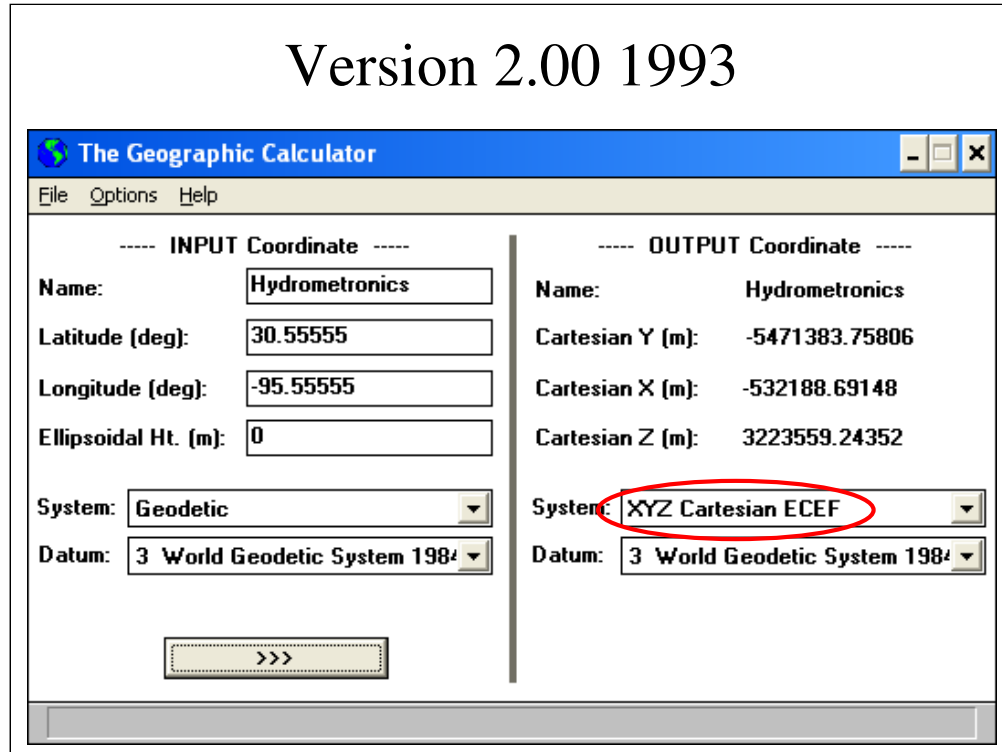
Let's step back a few years first. Here's version 2.00 of the Geographic Calculator in 1993, 17 years ago.   You can see that things were a whole lot simpler in 1993 … and less capable, too. But here's the same conversion from Geodeticlat/lon/height to XYZ Cartesian ECEF coordinates, with the same results, but with a different order of YXZ.
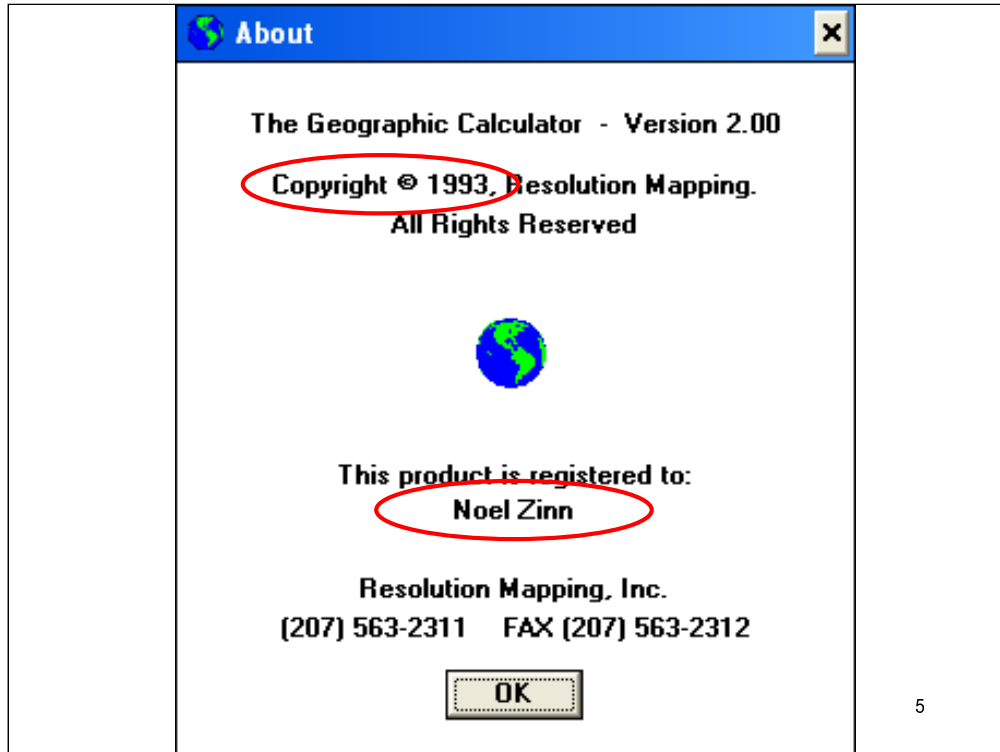
Here's the "About" screen from version 2.00 of the Geographic Calculator and I offer this just to establish my bona fides for giving this talk. I have been a registered user of the Geographic Calculator since 1993, even before Blue Marble was Blue Marble. It was Resolution Mapping at the time. But the blue marble logo was the same … and more about that later.

Geometronics, Inc. 1987
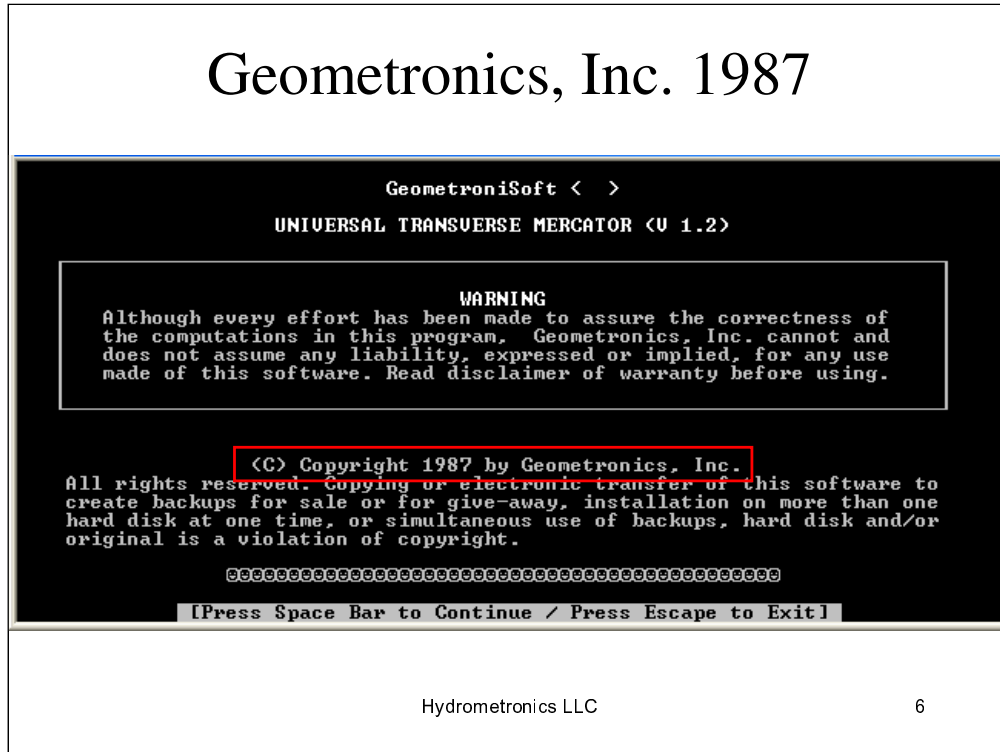
The Geographic Calculator had a profound effect on my career back in the late 1980s and early 1990s.

There was a slump in the oil industry in the late 1980s that was an opportunity for me to study programming. I took part time with my employer of the time and spent part time with my own company which I called Geometronics with the intention of writing and marketing geodetic and cartographic software.

There was a search on in the 1980s for a term to replace "surveying engineer" and "geometronics" was one of the replacement names in the ether. We all know that the winner of that search was another newly coined term "geomatics", which is used widely today. Anyway, I choose Geometronics as the name of my part-time company.

This slide is a capture from the splash screen of my UTM application in 1987. I had several other applications. In fact, I aspired to write The Geographic Calculator. But Resolution Mapping did that for me. So, I went back to my career in oil exploration and became a user of the Calculator, though I never gave up writing geodetic and cartographic software.

So, here's the home page of my Hydrometronics website today. Hydrometronics is obviously a nautical version of the name of my previous company, Geometronics, but it wasn't my first choice. That was Hydromatics, leveraging the term "geomatics" that emerged in the 1980s and 1990s. Unfortunately, a Dutch rock band called the The Hydromatics had that space covered. So, Hydrometronics was born in June of this year (2010) after I retired from my full-time career in oil exploration, most recently with ExxonMobil.

Nautical positioning is not my only professional interest. Geodetically-rigorous 3D visualization in ECEF coordinates is another, and that is the topic of this talk.

Notice that my website has it's own "blue marble".

EGM2008·10,000 in ECEF
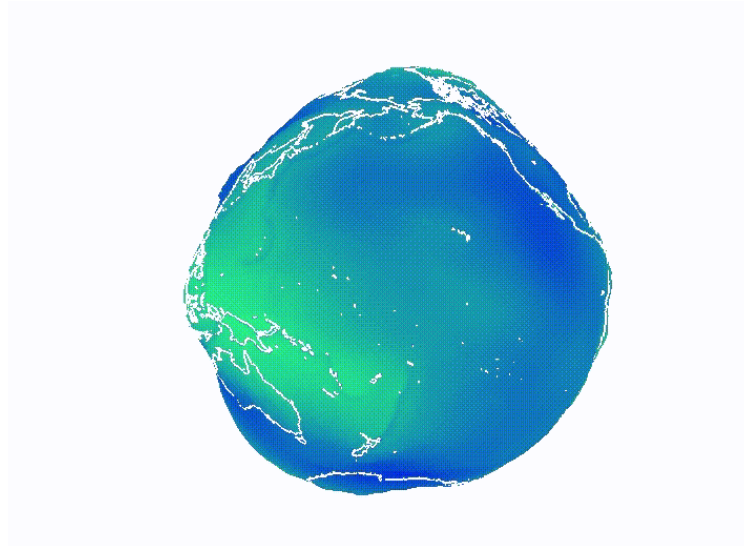
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That blue marble is an animated cartoon of the Earth Gravity Model 2008 exaggerated 10,000 times, depicted in ECEF, but its significance cannot be underestimated.

It's more than a pretty picture. First, it shows that the horizontal (the "flat" surface in which water settles) is neither flat nor even ellipsoidal. It undulates. Therefore, a 2D+1D perspective (projected coordinates with elevations attached), which assumes that the horizontal is flat, is misleading. Second, it shows that the gravity-based vertical dimension can easily be represented in ECEF. This is important for the integration of a vertical reference in ECEF.

Before one can represent the ECEF Earth in a visualization environment (VE) point elevations (the gravity-based vertical dimension) must be converted to heights (ellipsoid-based vertical dimension). EGM2008 is the best worldwide vertical model to use for this.
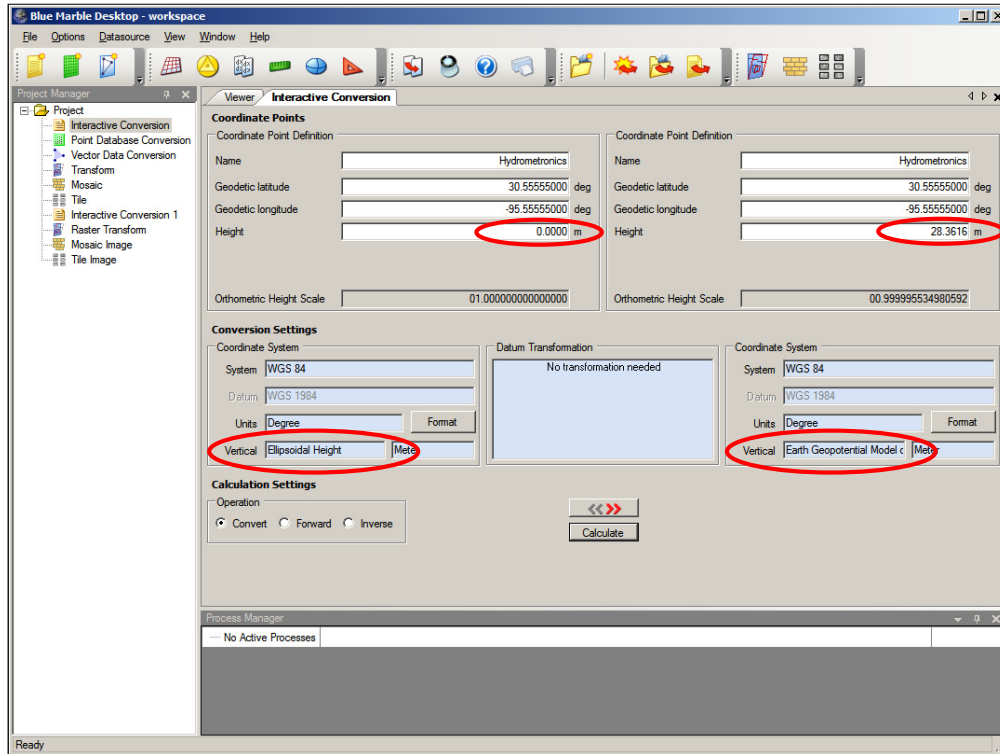
This BMGC exhibits the use of EGM2008. On the left we have an ellipsoidal height of 0.0 at the Hydrometronics home office. It's actually about +18m, but zero makes the math easy. On the right we get our vertical dimension in EGM, called Earth Geopotential Model in BMGC. The result is about 28.4m if you can read it. This mean that the EGM geoid is 28.4m above the WGS84 ellipsoid at my home office.

There are many models for the vertical dimension (that is, the geoid) and it is truly impressive that BMGC contains most, if not all, of them, a significant advancement over the 1993 version I showed you earlier. I began this talk by talking about an ECEF "scheme" for 3D visualization without distortion. BMGC's handling of vertical truly empowers this scheme.

EGM covers the entire world, so it is the geoidal model I prefer, but it is not the most accurate for North America. The next vertical datum for the United States (NAVD88's replacement) will be a geoid model. Ask Dave Doyle about Grav-D.

Overview

- Cartography (2D) is distorted.
- Geodesy (3D) is not, but …
- … 3D visualization environment (VE) required,
- … and geoid also required.
- Coordinate Reference System (CRS) primer
- Earth-Centered Earth-Fixed (ECEF)
- Topocentric coordinates (a "flavor" of ECEF)
- Orthographic projection (topocentric in 2D)
- This presentation => www.hydrometronics.com

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So, with these prefatory comments, here's our agenda for this afternoon.

(Read and elaborate.)

Cartography is distorted …
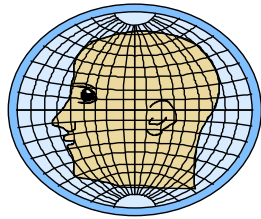
Globular projection
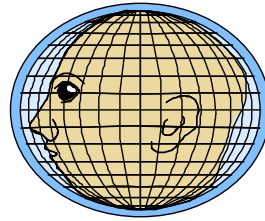
Orthographic projection
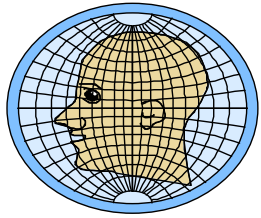
Stereographic projection
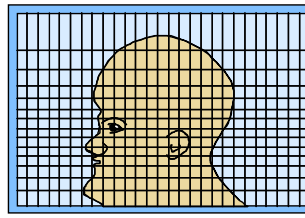
Mercator projection

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The simple point to be made with this slide is that 2D map projections necessarily change shapes in ways that are specific to the type of projection. Here are some examples. Both the Stereographic and the Mercator projections are conformal, which means that lines intersect at the same angle on the map that they do on the surface of the Earth. Local shapes are preserved on conformal projections, but large shapes change, and change differently (as can be seen). The Orthographic projection is the view from space (i.e. from infinity) and it plays an important role in the theme of this talk. More later on the Orthographic. The Globular projection is somewhere between the Stereographic and the Orthographic. Neither the Globular nor the Orthographic are conformal.

Cartography is distorted …

… but geodesy is not!

This graphic is of a latitude/longitude graticule and some low density satellite imagery in ECEF in a 3D geoscience visualization environment (VE). In ECEF the Earth is presented as an ellipsoidal that can be rotated with your cursor. Any particular area of interest can be viewed normally (that is, perpendicularly) without distortion. It's like having a globe in your hands.

A similar perspective is provided by Google Earth, but probably not in ECEF.

Google Earth's popularity has informed Earth scientists in the value of this perspective. A feature provided by ECEF in geoscience workstations that is not provided by Google Earth is the ability to view below the surface of the Earth into our seismic projects.
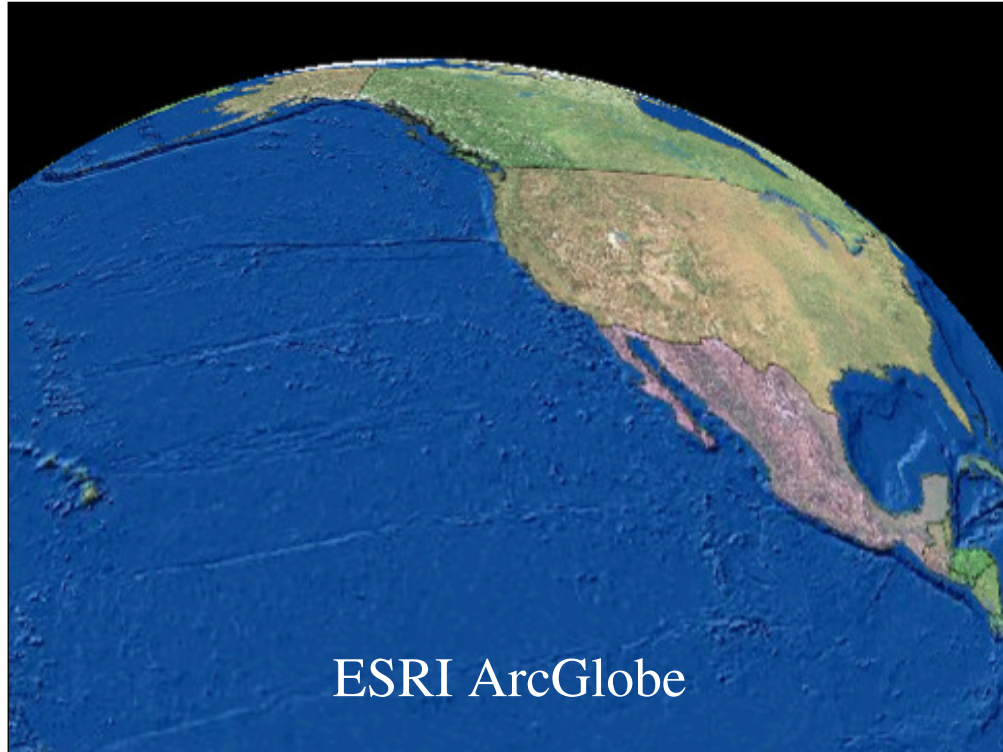
ESRI ArcGlobe

Another similar perspective is provided by ArcGlobe, a companion product to ESRI's ArcGIS.

ArcGlobe works its magic with a "cubic" projection, not with ECEF.

Issues

- Plate-to-pore scalability is desired in earth science software
- That software has heretofore used 2D projected coordinates in the horizontal and 1D depth/time in the vertical
- Projections have distortions of linear scale, area and azimuth that increase with project size
- These distortions can be quantified and managed on an appropriate map projection

These are the issues.

(Read and elaborate.)

Issues

- Earth science software is evolving toward visualization environments (VEs) that:
  - Operate in a 3D "cubical" CRS
  - Excel at graphical manipulation
  - Are geodetically unaware
- A pure 3D approach will:
  - Exploit the native power of VEs
  - Avoid the distortions (3D=>2D) of map projections
  - Achieve plate-to-pore scalability
  - Provide a new perspective on the data

(Read and elaborate.)

What are some VEs?

- Gocad (Paradigm, proprietary)
- Petrel, HueSpace (Schlumberger, proprietary)
- Matlab (The Mathworks, proprietary)
- ArcScene (ESRI, proprietary)

- VTK (Visualization Toolkit, open source)
- Mayavi (Python GUI front end to VTK, open source)

- Blue Marble? (some day!)
- iPod/Phone/Pad? Android? (some day, if not already!)

Let's unpack the phrase "visualization environment" (VE)

The largely French Gocad Research Group of multidisciplinary researchers (supported by a Consortium of 18 companies and 121 universities) began about 20 years ago to define new approaches to build and update 3D subsurface models. Today, the Gocad software is a commercial product of Paradigm Geophysical.

Petrel was developed by Technoguide in Norway using the HueSpace 3D renderer by Headwave, also Norwegian. Today both Petrel and HueSpace are owned by Schlumberger.

The Matlab "matrix laboratory" is my 3D VE of choice, using the 'plot3' command.

ArcScene (not ArcGlobe) is the ESRI 3D offering.

VTK is a $27M visualization environment (assessed by adding up developer hours) that you can download for free from www.vtk.org.

Mayavi is a user-friendly front end to VTK using the Python scripting language, an improvement of VTK's native Tcl/tk (tool command language/toolkit) IMO.

BMCG has some remarkable graphical capabilities in 2D, which is another significant advancement over the 1993 version I showed your earlier. Is 3D the next step?

I recently purchased an iPod touch because it has an inertial measurement unit (IMU) onboard. Some of the iPod apps certainly look 3D to me. How about ECEF in an iPod?

Heritage Applications
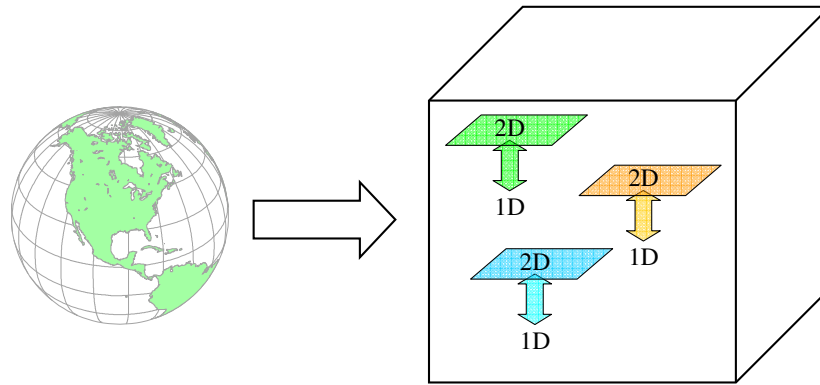
Heritage earth-science applications with internal geodesy support any projected coordinate system (2D horizontal + 1D vertical), but with the usual, well-known mapping distortions

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Examples of heritage, geodetically-aware geophysical applications are Schlumberger's GeoFrame and Landmark's OpenWorks (which has BMGC as its geodetic engine). Multiple 2D projections and multiple datums coexist side by side in these applications. Projects can be transformed from datum to datum or converted from projection to projection as data management requirements dictate. Projection distortions can be managed in such as system, but distortion is always there nevertheless. The horizontal dimension is presumed to be flat with the vertical dimension perpendicular to the horizontal.

Current Path to VE via Middleware

3D World   Middleware   VE in 2D+1D
Projection

3D=>2D

2D

1D
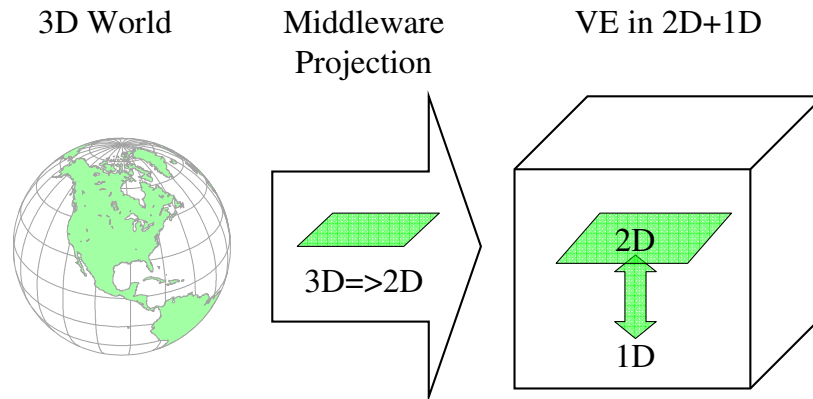
VEs have no internal geodesy. Coordinates are projected "outside the box" (in middleware). Only one coordinate system is allowed inside the box at a time.

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Examples of geoscience visualization environments (VE) are Petrel and GoCAD. Only one datum and projection lives inside a VE at any one time. Projection distortions cannot be managed in a VE, which is best suited to reservoir-sized prospects (minimal distortion). Regional studies have large projection distortions.

Update: Petrel projects can be flushed from the VE and reloaded in a different projection or datum as data management requirements dictate.

An example of middleware is TIBCO OpenSpirit.

Proposed Path to VE via ECEF

3D World   VE in true 3D
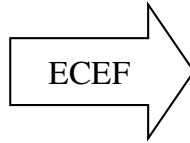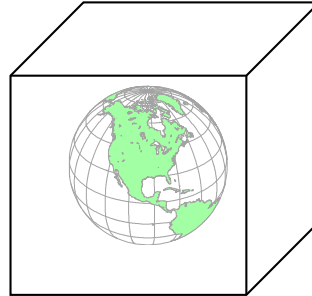
ECEF

If ECEF coordinates are chosen in middleware, the VE
"sees" the world in 3D without any mapping distortions. If
ECEF coordinates in WGS84 are chosen, then projects
throughout the world will fit together seamlessly.

This slide depicts the (perhaps) revolutionary step proposed in this presentation.
That is, use the ECEF coordinate system (described later) to move a 3D Earth into a
3D visualization environment (VE). Geodetic rigor is maintained. There is no
projection distortion. Each prospect can be worked locally. All projects fit together
globally. A VE in ECEF is suitable for both local and regional projects.

EPSG Coordinate System Primer

1. Geographical 2D (lat/lon) and Geographical 3D (lat/lon/height with respect to the ellipsoid)
2. Vertical (elevation or depth w.r.t. the geoid)
3. Projected (mapping of an ellipsoid onto a plane)
4. Engineering (local "flat earth")
5. Geocentric Cartesian (Earth-Centered Earth-Fixed)
6. Compound (combinations of the above)

These are the coordinate reference systems (CRS) described by the Surveying and Positioning committee of the International Association of Oil and Gas Producers (OGP), formerly the EPSG.

Geographical CS: lat/lon/(height)

A graticule of curved parallels and curved meridians (latitudes and longitudes) intersect orthogonally on the ellipsoid. Height is measured along the normal, the straight line perpendicular to the ellipsoid surface.

No notes.

Vertical CS: elevation
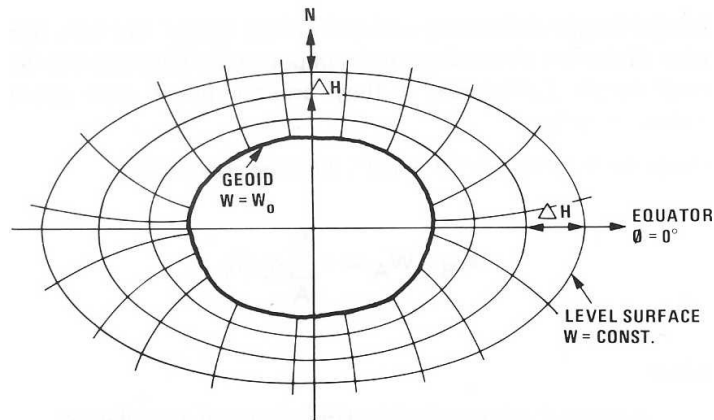
Elevation is measured along the (slightly curved) vertical, which is perpendicular to the irregularly layered geopotential surfaces of the earth. The geopotential surface at mean sea level is called the geoid. (Graphic from Hoar, 1982.)

No notes.

Projected CS: Northing/Easting

- Map projections of an ellipsoid onto a plane preserve some properties and distort others
  - Angle - and local shapes are shown correctly on conformal projections
  - Area - correct earth-surface area (e.g., Albers)
  - Azimuth - can be shown correctly (e.g., azimuthal)
  - Scale - can be preserved along particular lines
  - Great Circles - can be straight lines (Gnomonic)
  - Rhumb Lines - can be straight lines (Mercator)
- **Rule of thumb: map distortion $\propto$ distance^2**

A map projection is a mathematical "mapping" of 3D ellipsoidal space onto a 2D planar space. Distortions are inevitable. But we can preserve selected properties of the 3D surface by our choice of mapping equations.

In this slide I've listed some of the desirable preservations.

We can preserve some features, but will unavoidably distort other features.

Distortions increase proportionally to the square of the distance.

Projected CS Distorts
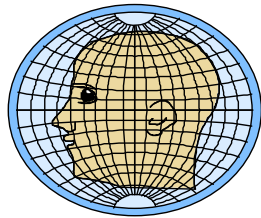
Globular projection
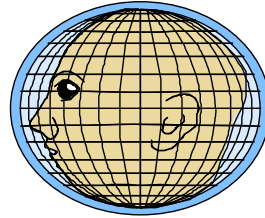
Orthographic projection
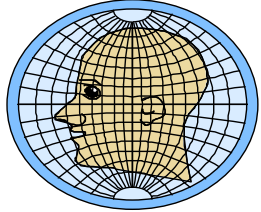
Stereographic projection
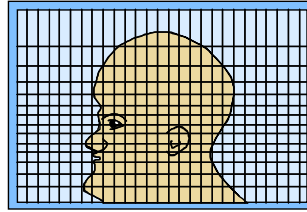
Mercator projection

Rule of thumb: map distortion $\propto$ distance^2

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Not only do different projections depict shape differently, but re-projection from one projection to another (even if conformal) changes shape.

Engineering CRS ("Flat-Earth")
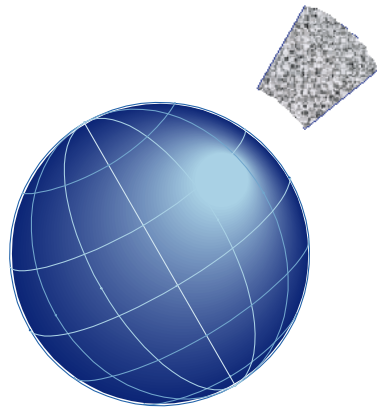
Our project extracted
from an ellipsoidal earth
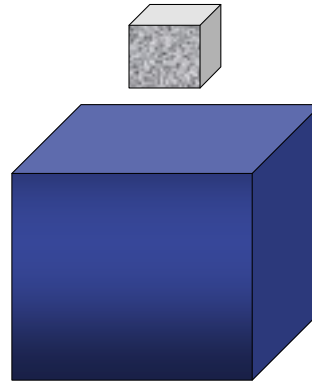
Our project extracted from
a cubical, flat earth

The Engineering CRS presents the world as a cube, which is an approximation valid only over a small, local area. Nevertheless, this cubical concept permeates our thinking about our projects over larger areas. For example, geophysical data processing presumes that all verticals are parallel. In fact, verticals converge.

Geocentric CRS (ECEF)
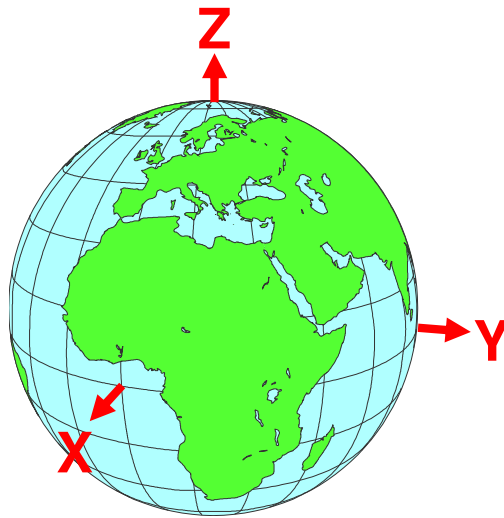
The Z-axis extends from the geocenter north along the spin axis to the North Pole. The X-axis extends from the geocenter to the intersection of the Equator and the Greenwich Meridian. The Y-axis extends from the geocenter to the intersection of the Equator and the 90E meridian.

Earth-Centered Earth-Fixed (ECEF) is also known as Geocentric CRS. Any point on or near the surface of the earth is represented in a 3D, rectilinear, right-handed XYZ coordinate frame fixed to the Earth. The origin (0, 0, 0) is the geocenter. The positive X-axis extends from the geocenter through the intersection of the Greenwich Meridian with the Equator. The positive Y-axis extends from the geocenter through the intersection of the 90E meridian with the Equator. The positive Z-axis extends from the geocenter through the North Pole.

Coordinate Conversion

- The mathematics of map projections (3D=>2D) are complicated (especially TM) and generally valid over limited extents
- The mathematics of converting Geographical CS coordinates to ECEF Geocentric CS (3D=>3D) are simple and valid the world over
- Blue Marble Geographic Calculator handles them both

The validity of map projections are constrained in two ways. First, distortions increase as the square of distance. Second, the algorithmic implementation of some projections (especially the Transverse Mercator) introduces computational errors as one moves from the center or central meridian of the projection.

The geographical ⇔ geocentric (ECEF) conversion does not suffer this problem.

So, Why ECEF?

- ECEF is the geodetic CS native to a 3D VE
- ECEF in a 3D VE is a globe in your hands
- Given the proper perspective (turning the globe), ECEF coordinates have no distortion
- ECEF is scalable from plates to pores
- No geodetic "smarts" are required in the VE

No notes.

North America in VTK
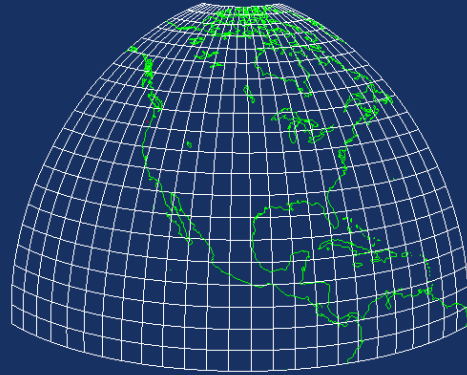
This demo is not available in the PDF version of this presentation. It shows a cartoon of the North American octosphere. The image is rotated to show distortion-free perspectives wherever desired.

U.S.G.S. Coastline Culture
Excerpts in Geographical and ECEF

Geographical CS (height = 0)		Geocentric CS (ECEF)		
longitude	latitude	X	Y	Z
NaN	NaN	NaN	NaN	NaN
−50.027484	0.957509	4096874.92	−4887224.49	105871.03
−50	0.99249	4099176.47	−4885208.29	109738.48
NaN	NaN	NaN	NaN	NaN
−59.708179	8.277287	3183867.68	−5450322.48	912137.99
−59.773891	8.310143	3177350.79	−5453517.54	915733.77
−59.905313	8.462687	3163599.63	−5458662.31	932424.41
NaN	NaN	NaN	NaN	NaN
−57.060949	5.791989	3450502.62	−5325702.36	639376.55
−57.117273	5.90229	3444590.92	−5328048.22	651510.81
−57.161863	6.066569	3439416.28	−5329135.93	669578.81
−57.272164	6.26605	3427869.60	−5333753.93	691511.19
−57.391853	6.308293	3416444.41	−5340472.04	696154.65
−57.546744	6.442062	3401113.29	−5348302.30	710856.40

Here's what ECEF coordinates look like. This is coastline culture downloaded from NOAA (link at the end of this presentation) in Matlab format.

On the left are latitude and longitude. We assume that height is zero. The NaNs mark the beginning and end of connected polygons. Matlab interprets these as "lift pen" commands.

On the right are ECEF XYZ for some small part of North America.
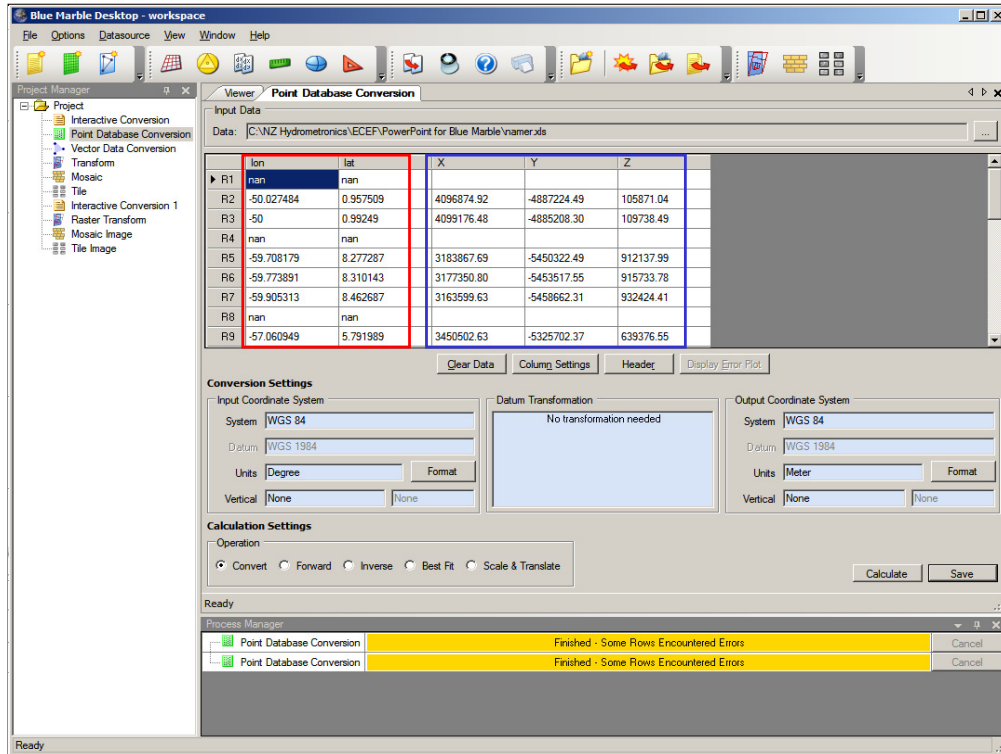
Here's the same geodetic to ECEF conversion done in batch in BMGC.

We have geographical lat/lon in red and ECEF XYZ in blue.

Translation & Rotation:
ECEF to Topocentric

- A journey back to Projected CS because …

- … some users may prefer their data referenced to their local area of interest

- ECEF can easily be translated and rotated to a topocentric reference frame

- This conversion is conformal,
  it preserves the distortion-free curvature of the earth,
  and the computational burden is small

- VEs already do something similar to change the viewing perspective

This slide marks an important transition in the presentation, the translation and rotation from geocentric ECEF coordinates to topocentric coordinates, called East/North/Up (ENU) in Wikipedia, topocentric horizon by Bugayevskiy & Snyder, local vertical by the Manual of Photogrammetry and local horizontal by myself previously.

ECEF coordinates present the whole world – or just your local project – from the geocentric perspective. The geocenter may be far away. The geoscientist may prefer a local origin for their project. That is provided by topocentric coordinates (called UVW here), which preserve all the curvature of the Earth. But the perspective is local and more familiar.

EPSG Graphic of Topocentric

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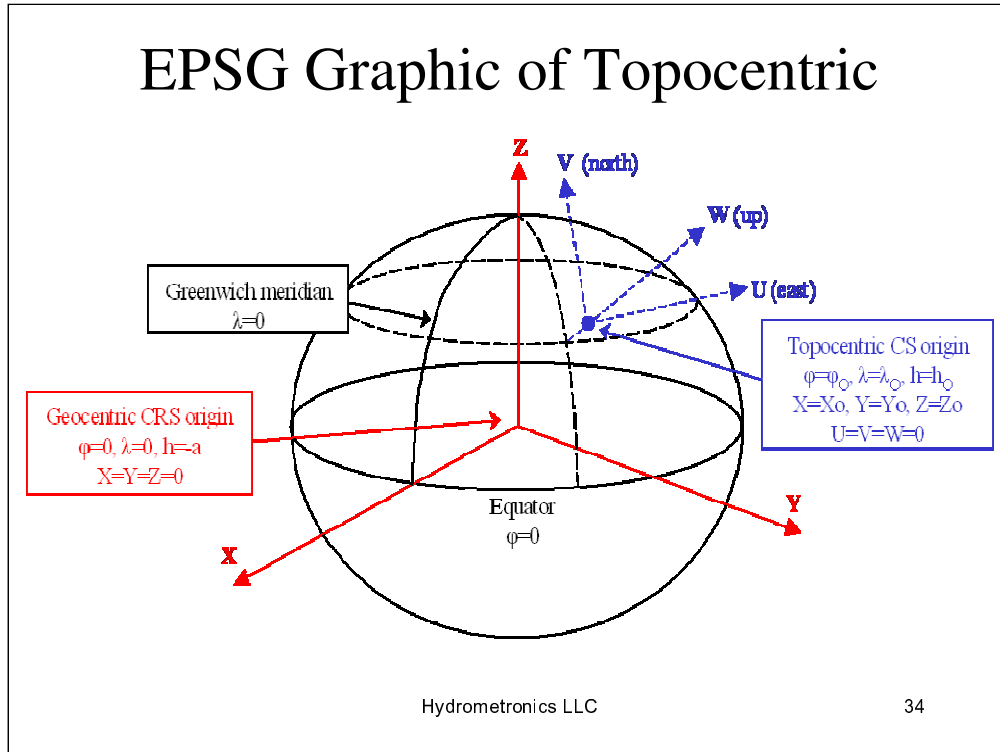
ECEF (XYZ) is shown in the red coordinate frame, topocentric (UVW) in the blue. A translation and a rotation are required to convert one into the other. These equations are well-known and can be found in the EPSG Guidance Note 7 Part 2 (www.epsg.org).

GOM in Topocentric Coordinates
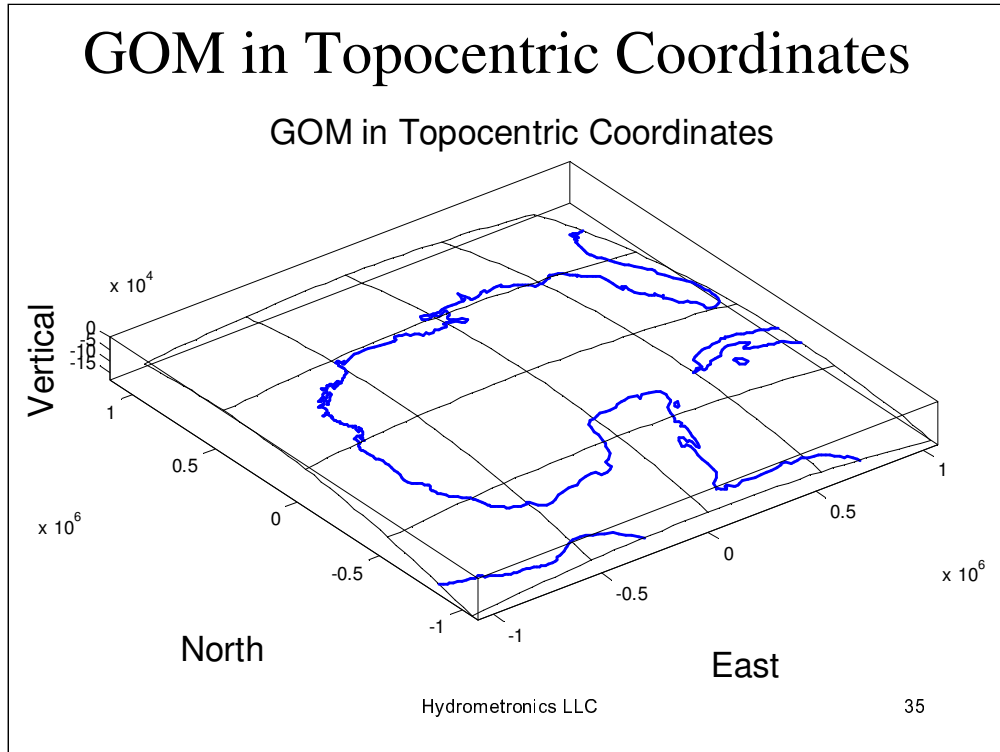
Here's the entire GOM shown in 3D topocentric coordinates. The curvature of the Earth is still visible, just not as much of it. The more local one becomes, the less curvature one sees.

Topocentric to Orthographic

- Continuing the journey
- The orthographic projection is the view from space, e.g. our view of the moon
- Topocentric without the W vertical coordinate (3D=>2D) is the Orthographic projection
- The ellipsoidal Orthographic projection is a bona fide map projection with quantifiable distortions intermediate between our usual 2D+1D paradigm and a new topocentric / ECEF 3D paradigm

This slide marks a second important transition, that from 3D topocentric coordinates to 2D orthographic. The transition is simple. U (of UVW) becomes Easting, V becomes Northing, and W goes away.

Our view of the moon is orthographic.

Orthographic Projection of GOM
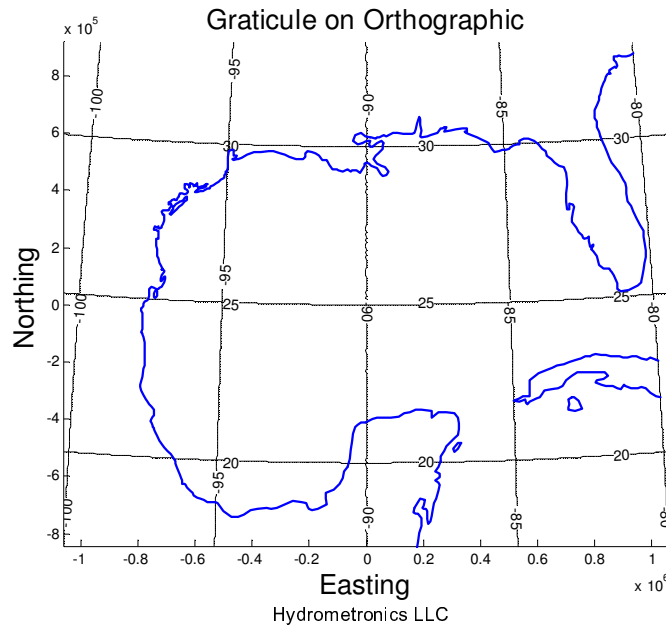
Here's the GOM shown previously in 3D topocentric coordinates now represented in 2D orthographic (projection) coordinates.

This is a projection with quantifiable (and thus manageable) distortions. The orthographic is neither conformal nor equal area, but near the center distortion is negligible.

Oblique Ellipsoidal Orthographic Minimum Scale

This is scale in the radial direction. Scale in the circular direction is 1.0000
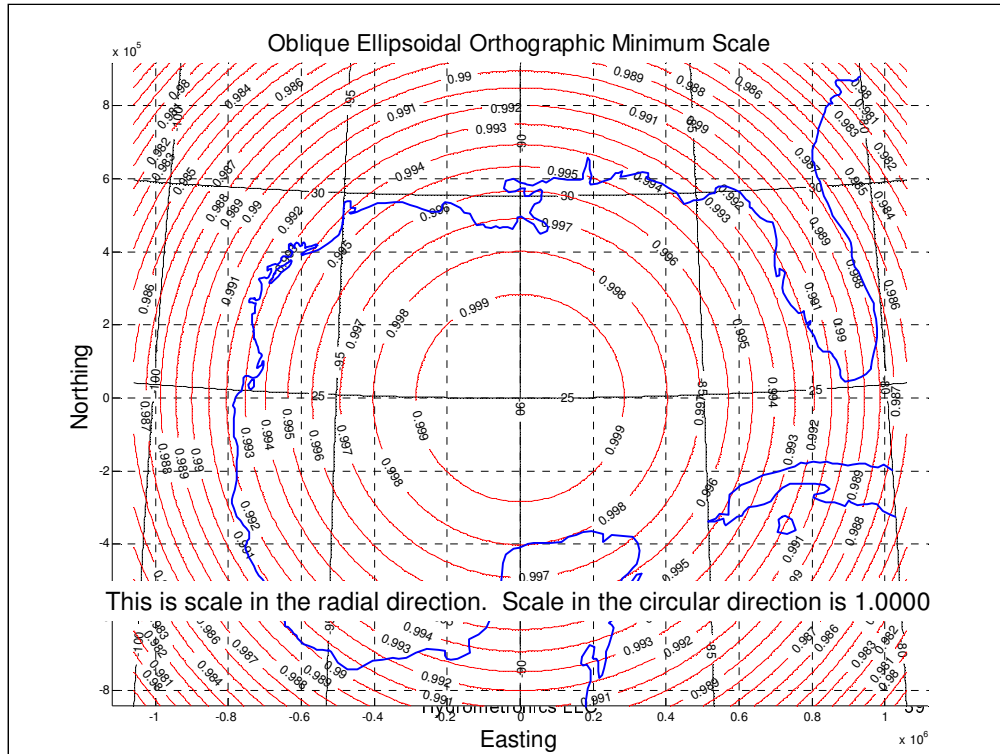
This graphic depicts scale distortion on the ellipsoidal orthographic. There is no scale distortion (scale = 1) in the direction perpendicular from a point to the center of the projection. In the direction from a point to the center it is that shown on this graphic. Within 90km of the origin the minimum scale is less than 1 part in 10,000. Within 180km of the origin the minimum scale is less than 4 parts in 10,000 (about that of TM within a UTM zone).

If one needs to work within the 2D+1D paradigm, then consider the Orthographic projection. It's one dimension away from topographic, which is a rotation and a translation away from ECEF.
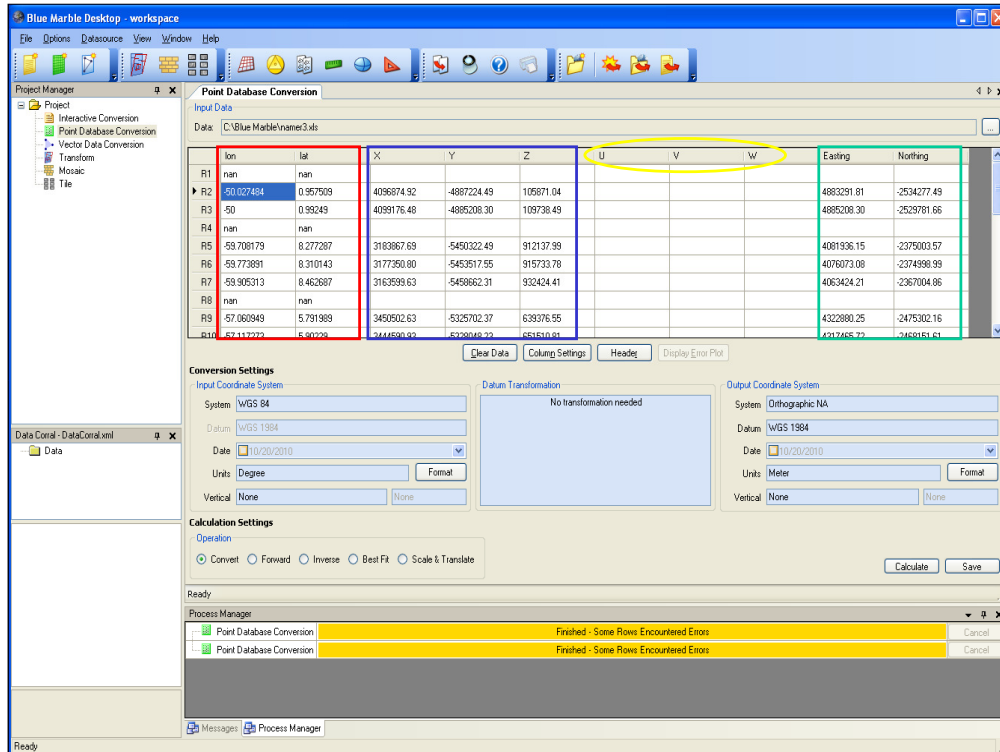
And here's the conversion of NOAA coastline culture from geodetic to ECEF to Orthographic projected coordinates in BMGC.

Again, geographical lat/lon are in red, ECEF XYZ are in blue, and Ellipsoidal Orthographic Northing/Easting are in green in the beta-test version of BMGC calculator that I'm using and which may be released as version 7.6.

What's missing are the topographic UVW columns in yellow. These are currently being programmed into BMGC and will be available soon, the last piece of this ECEF scheme for 3D visualization without distortion.

But I can tell you that we already know the topocentric U and V columns. Those will be the Easting and Northing of the ellipsoidal orthographic. We're only waiting on W!

Our Journey Schematically

Proj 2D+1D
Northing
Easting
Elevation

→ 1 / Proj →

Geog 2D+1D
Latitude
Longitude
Elevation

→ EGM →

Geog 3D
Latitude
Longitude
Height

↓ ECEF

Orthographic
Northing
Easting
(Vertical)

← U=E, V=N ←

Topocen 3D
U
V
W

← Δ + Θ ←

Geocen 3D
X
Y
Z

**All the undistorted curvature
of the Earth in a 3D VE**
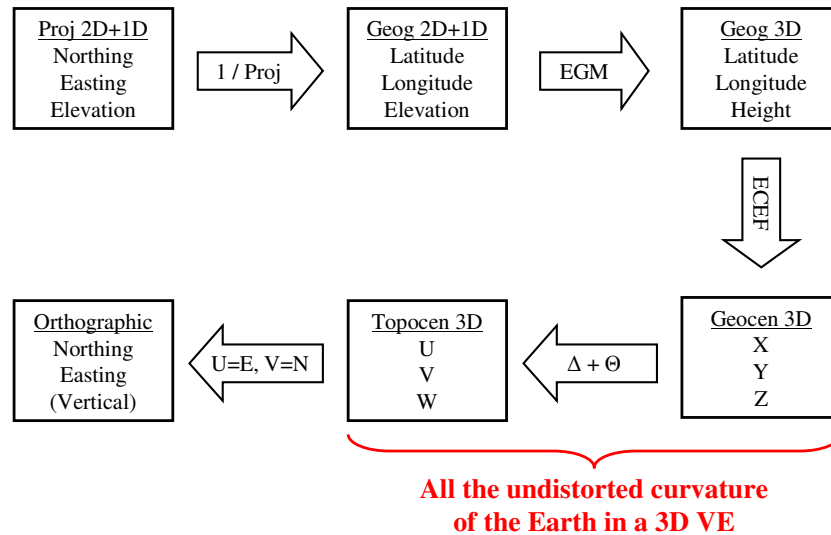
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We've been on a journey this afternoon and this slide presents that journey schematically.

We begin with the 2D+1D geodesy and cartography (G&C) of heritage workstations in the upper left, namely, projected coordinates in the horizontal and a geoid-based elevation in the vertical.

Then we use an inverse projection to get to a geographical 2D+1D G&C (latitude / longitude / elevation).

Then we use EGM2008 to get to a truly 3D coordinate system (latitude / longitude / ellipsoid-based height). But this 3D CS is not one that fits well into a VE.

Then we take the biggest step of all from Geographical 3D to Geocentric 3D (or ECEF). At this point all the undistorted curvature of the world fits into a geodetically unaware 3D visualization environment.

The next step localizes the perspective with a translation (Δ) and a rotation (Θ) to a topographic origin while still preserving all the undistorted curvature of the world.

These two steps are our desired destination.

 But topocentric 3D is just an extra dimension (W) added onto the Ellipsoidal Orthographic projection, a serviceable piece of cartography.

Our Journey Schematically
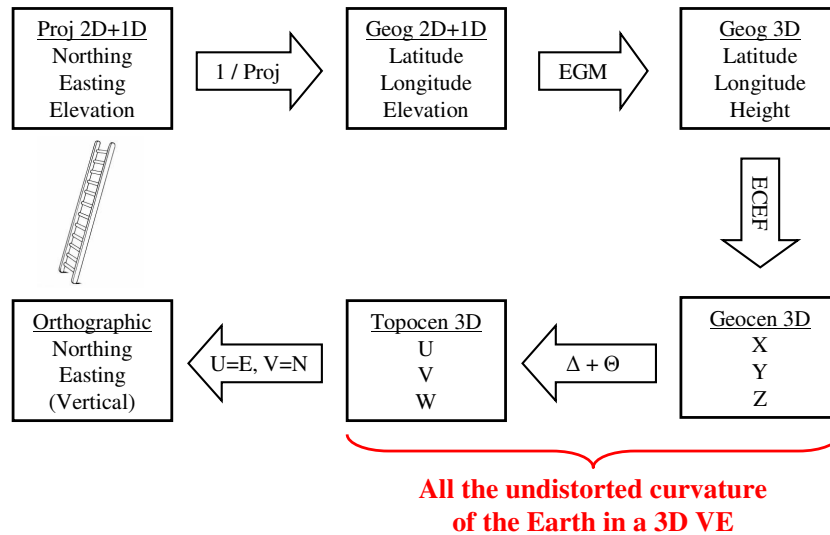
And here we have a ladder uniting us with the beginning of our journey, a 2D map projection with manageable distortions that may be "good enough for seismic".

Summarizing

- Cartography (2D) is distorted; geodesy (3D) is not
- Not all 3D presentations are ECEF (geodesy)
- Geodetically "unaware" visualization environments (VE) present an opportunity
- Coordinate Reference System (CRS) primer
- **Earth-Centered Earth-Fixed (ECEF)**
- **Topocentric coordinates (a "flavor" of ECEF)**
- **Orthographic coordinates (2D topocentric)**
- Blue Marble is committed to supporting this scheme

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(Read and elaborate.)

Conclusion

- The real world is 3D
- New visualization environments are 3D
- Why incur the distortions of a 2D map projection entering real-world data into a VE?
- ECEF, topocentric and orthographic coordinates are a paradigm shift in the way we view our data, a perspective that may extract new information
- It's time for ECEF … and Blue Marble is there!

No notes.

More Information

- This presentation can be downloaded at www.hydrometronics.com
- There is a ECEF Group on LinkedIn
- Guidance Note 7-2 at www.epsg.org
- Wikipedia (search ECEF)
- World coastlines are available at www.ngdc.noaa.gov/mgg/shorelines/shorelines.html

No notes.

Extra Slides

Mini Bio of Noel Zinn

- Noel Zinn began Hydrometronics LLC in 2010 as a technical software consultancy
- Geodesist for ExxonMobil in the Naughts
- Navigation Scientist for Western Geophysical in the Nineties
- Surveyor for NCS International in the Eighties
- Navigator for Delta Exploration (Singapore) in the Seventies
- Peace Corps Volunteer in India in the Sixties
- Studied at the University of California (Berkeley) and the University of Houston (still there!)

Noel Zinn's professional bio.

Geographical to ECEF Coordinates

Given the ellipsoid semi-major axis (a) and flattening (f), and latitude (ϕ), longitude (λ), and height (h)

$$b = a - a \cdot f \qquad e^2 = (a^2 - b^2)/a^2 \qquad \nu = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}}$$

$$X = (\nu + h) \cos \phi \cos \lambda$$

$$Y = (\nu + h) \cos \phi \sin \lambda$$

$$Z = (\nu(1 - e^2) + h) \sin \phi$$

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Intermediate terms are the semi-major axis (b), eccentricity squared (e^2) and the radius of curvature in the meridian (nu).

This conversion is exact.

ECEF to Geographical Coordinates

Given ellipsoid a and f, and X, Y and Z Cartesians

$$b = a - a \cdot f \qquad e^2 = (a^2 - b^2)/a^2 \qquad e'^2 = (a^2 - b^2)/b^2$$

$$v = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}} \qquad p = (X^2 + Y^2)^{1/2} \qquad \theta = \tan^{-1}(\frac{Z \cdot a}{p \cdot b})$$

$$\phi = \tan^{-1} \frac{Z + e'^2 b \sin^3 \theta}{p - e^2 a \cos^3 \theta}$$

$$\lambda = \tan^{-1}(\frac{Y}{X})$$

$$h = (p/\cos\phi) - v$$

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Intermediate terms are the semi-major axis (b), eccentricity squared (e^2), eccentricity prime squared (e'^2), the radius of curvature in the meridian (nu), the projection of the point on the Equatorial plane (p) and theta.

This conversion is acceptably precise within any working distance of the surface of the Earth.

U.S.G.S. Coastline Culture
Excerpts in ECEF and Topocentric

Geocentric CRS (ECEF)			Topocentric		
X	Y	Z	U-East	V-North	W-Up
NaN	NaN	NaN	NaN	NaN	NaN
4096874.92	-4887224.49	105871.03	4883291.81	-2534277.49	-3159278.92
4099176.47	-4885208.29	109738.48	4885208.29	-2529781.65	-3158620.16
NaN	NaN	NaN	NaN	NaN	NaN
3183867.68	-5450322.48	912137.99	4081936.14	-2375003.57	-2094765.47
3177350.79	-5453517.54	915733.77	4076073.08	-2374998.99	-2089176.88
3163599.63	-5458662.31	932424.41	4063424.20	-2367004.86	-2072737.89
NaN	NaN	NaN	NaN	NaN	NaN
3450502.62	-5325702.36	639376.55	4322880.24	-2475302.15	-2399575.74
3444590.92	-5328048.22	651510.81	4317465.71	-2468151.60	-2389219.87
3439416.28	-5329135.93	669578.81	4312558.56	-2455576.85	-2376097.067
3427869.60	-5333753.93	691511.19	4301989.21	-2442987.79	-2356979.39
3416444.41	-5340472.04	696154.65	4291904.18	-2444958.68	-2347406.64
3401113.29	-5348302.30	710856.40	4278165.68	-2440364.45	-2330009.96

On the left are the ECEF XYZ for some small part of North America that we've
seen already. On the right are the topocentric equivalents for an origin of
40N/100W.

U.S.G.S. Coastline Culture Excerpts in Topocentric and Orthographic

Topocentric			Orthographic	
U-East	V-North	W-Up	Easting	Northing
NaN	NaN	NaN	NaN	NaN
4883291.81	−2534277.49	−3159278.92	4883291.81	−2534277.49
4885208.29	−2529781.65	−3158620.16	4885208.29	−2529781.65
NaN	NaN	NaN	NaN	NaN
4081936.14	−2375003.57	−2094765.47	4081936.14	−2375003.57
4076073.08	−2374998.99	−2089176.88	4076073.08	−2374998.99
4063424.20	−2367004.86	−2072737.89	4063424.20	−2367004.86
NaN	NaN	NaN	NaN	NaN
4322880.24	−2475302.15	−2399575.74	4322880.24	−2475302.15
4317465.71	−2468151.60	−2389219.87	4317465.71	−2468151.60
4312558.56	−2455576.85	−2376097.067	4312558.56	−2455576.85
4301989.21	−2442987.79	−2356979.39	4301989.21	−2442987.79
4291904.18	−2444958.68	−2347406.64	4291904.18	−2444958.68
4278165.68	−2440364.45	−2330009.96	4278165.68	−2440364.45

Here are the topocentric data we've seen before on the left and the equivalent orthographic data on the right. Orthographic projection coordinates are just topocentric coordinates without the vertical value.